Central Banks Avoid Reporting Losses Through Foreign Exchange Interventions

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Abstract

While recent studies show that central banks avoid reporting losses, how they achieve this is unclear. This paper reveals that central banks avoid reporting losses through foreign exchange interventions (FXI) and demonstrates how such loss-avoiding behavior can lead to welfare gains. I show that central banks perform FXI that increases their profits right before releasing financial statements, and the magnitude of these interventions varies predictably with central banks' incentives to avoid losses. These interventions are welfare-reducing in ordinary circumstances. However, I demonstrate that when the nominal interest rate is at the zero lower bound, central banks' loss-avoiding behavior *can* be welfare-increasing; it can serve as a commitment device and provide an optimal escape from the liquidity trap.

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[A negative profit] could be seen by some as a sign of mismanagement of "taxpayer resources" by the Federal Reserve and might well invite the scrutiny of Congress in a way that could be damaging to the Federal Reserve's reputation and independence.

- Federal Open Market Committee (FOMC) Memorandum, 2013.

Does the Riksbank (Swedish central bank) have to make a profit? Yes, it does. ... We need to be able to build up buffers to cover our costs so that we can carry out our task regardless of the government and the parliament.

- Kerstin af Jochnick (First Deputy Governor of the Riksbank), 2015.

1 Introduction

Central banks tend to avoid reporting losses amid external pressure. Goncharov et al. (2023) document that central banks worldwide are discontinuously more likely to report slightly positive profits than slightly negative ones, especially when political pressure is greater and central bank governors are eligible for reappointment. They conclude that central banks avoid reporting losses due to agency problems. Rogoff (2016) also sheds light on this issue by pointing out how central banks' independence can be linked to their profits. The motives and evidence that central banks tend to avoid reporting losses are clear. However, two questions remained unaddressed in the literature. First, *how* do central banks avoid losses? Second, what are the welfare implications for such loss-avoiding behavior? This paper addresses these two questions.

The two questions are important since there are potentially many ways central banks can avoid reporting losses. If central banks only manage profit through accounting practices, it would have little impact on the real economy. The same cannot be said if central banks use policy tools to avoid reporting losses. In this case, profit concerns could impact monetary policies and have essential welfare implications. The questions are also timely since many central banks adopted quantitative easing (or large-scale asset purchases) during COVID-19. Central bank profits, therefore, are increasingly sensitive to the economic environment and monetary policies. Moreover, central banks worldwide have accumulated large foreign reserves over the past few decades, further exposing them to exchange rate risks. This paper focuses on revealing the methods used by central banks to avoid losses and their welfare implications.

First, using a novel dataset of 116 central banks' financial statements from 2000 to 2024, I document that central banks avoid reporting losses. The distribution of central bank profits shows that banks are discontinuously more likely to report small positive profits than small negative ones. In a standard central banking model, profits should be irrelevant, and their distribution should be smooth. The discontinuity at zero profit serves as evidence of profit management performed by central banks. Goncharov et al. (2023) first discover the discontinuity in central banks' profit distribution. Using a different dataset, the results in this paper are consistent with their findings.

Next, I demonstrate that central bank profits increase when their local currency depreciates. Central banks holding foreign reserves (assets denominated in foreign currencies) must still prepare their financial statements in local currencies, regardless of the accounting rule used. Therefore, all foreign reserves will be evaluated in terms of the local currency when the financial statements are released. Local currency depreciation would naturally lead to a re-evaluation gain on the reserves; the larger the foreign reserves, the greater the gain.

This profit channel is economically significant for central banks worldwide. This study covers 1,906 bank-year observations on annual balance sheets from 116 central banks. The net foreign asset (foreign asset minus foreign debt) to total asset ratio for the median observation is 66%, while the first and third quartiles are 41% and 84%, respectively. Given that central banks worldwide generally hold large amounts of foreign reserves, a small fluctuation in the exchange rate can greatly impact the reported profit.

The first key contribution of this paper is to show that central bank profit concerns affect monetary policies. Specifically, this research provides evidence that profit motives impact how central banks intervene in the foreign exchange market. This is the first research to establish the causal impact of profit concerns on monetary policies. As noted previously, the depreciation of local currencies leads to profits through the re-evaluation gains on foreign reserves. Therefore, the exchange rate at the last fiscal month, when profits are being reported, becomes essential for central banks with profit concerns. The intervention pattern revealed by this paper is intuitive—central banks intervene and depreciate their local currencies right before reporting profits, pointing to a causal relationship between profit concerns and such interventions.

To establish the causal relationship, I obtain information on central banks' timing of profit reporting (their last fiscal month) and data on foreign exchange interventions (FXI). The information on the last fiscal month for the 116 central banks under study is obtained manually from their financial statements or websites. For instance, the Bank of England's last fiscal month is February, March for the Monetary Authority of Singapore, June for the Reserve Bank of Australia, and December for the Sveriges Riksbank. On the other hand, I employ monthly FXI measurements based on Adler et al. (2024), which refines the traditional proxy. The traditional FXI measurement is constructed using the changes in the stock of the central bank's foreign reserves (measured in USD). Adler et al. (2024) improve upon it by including a more comprehensive range of central bank operations (such as derivative transactions) and adjusting for valuation changes, income flows, and changes in other foreign-currency balance sheet positions. With the refined measurement of FXI and the knowledge about the time of reporting, this paper shows that interventions by central banks to depreciate local currencies are particularly likely in their *last fiscal month*. Central banks intervene to increase profit right before releasing financial statements, suggesting that profit concerns cause these interventions.

The first concern for the causal identification is seasonality. Although central banks can choose any month as their fiscal year-end, around 80% of them choose December (see Table 1 for details). As a result, the aggressive FXI at the fiscal year-end to depreciate local currency (henceforth, "the distortion pattern") could be due to seasonality, and the profit increase is simply an unintended byproduct. The other concern for the identification is the possibility that the distortion pattern is due to profit-unrelated reasons. It may be possible for the central bank to intervene in the last fiscal month per the central government's request, to boost trade, or for other reasons that are unrelated to profit.

I rule out these concerns by showing that the significance and magnitude of the distortion pattern varies predictably with central banks' *incentives* to avoid losses. Suppose the distortion pattern is, in fact, due to seasonality or profit-unrelated reasons. In that case, the pattern should be observed regardless of the central bank's incentive to avoid reporting losses. However, this research shows that the pattern is only observed for central banks with strong incentives to avoid losses and not observed for those with little or no incentives.

I first show that the distortion pattern is more significant when central banks face greater financial pressure, particularly when they reported losses last year. The same distortion pattern is not observed for central banks that reported a large profit the previous year. Moreover, the pattern is only observed for central banks with large foreign reserves. Exchange rate movements have minimal effect on profit for central banks with little foreign reserves. Therefore, they have no incentive to depreciate for profit motives, and indeed, the distortion pattern is not observed for central banks with little foreign reserves. Finally, as previously mentioned, one key incentive for central banks to avoid losses is to protect their independence. The data verifies this. Legally independent central banks are shown to be more likely to display the distortion pattern compared to the central banks that are more integrated with the government.

These findings show that the magnitude and significance of the distortion pattern depend on how incentivized the central banks are regarding loss avoidance. The results also support the conclusion that these interventions are motivated by profit concerns.

The other key contribution of this paper is to provide welfare analysis for central banks' loss-avoiding behavior. This paper demonstrates that such profit concerns are generally welfare-reducing theoretically. However, when the nominal interest rate is at the zero lower bound, central banks' loss-avoiding behavior *can* be welfare increasing—it can serve as a commitment device and provide an optimal escape from the liquidity trap.

This paper builds on a standard small open economy New Keynesian model from Galí (2015) with two additions. First, a zero lower bound (ZLB) on the nominal interest rate is introduced. Second, the model includes an independent central bank with flow profits and asymmetric objectives. The foreign reserves' re-evaluation gains or losses determine the central bank's flow profit. The central bank's asymmetric objectives are designed as follows: When the flow profit is positive, the central bank has standard objectives that aim to minimize inflation and output gap. On the other hand, when the profit is negative, the central bank aims to minimize inflation, output gap, *and losses*. This asymmetric preference captures the fact that central banks are not profit maximizers. However, they do try to avoid losses.

The model is linearized around a zero-inflation steady state and solved using the local perturbation method. To deal with the non-linearity caused by the ZLB on the interest rate and the asymmetric objectives, this research utilizes the piece-wise linear solution for occasionally binding constraints provided by Guerrieri and Iacoviello (2015).

The model shows that the central banks' loss-avoiding behavior can help the economy escape a liquidity trap. It is useful first to note that the nominal interest rate hits zero during a liquidity trap while the real interest rate is still higher than optimal. It is well known since Krugman (1998) and Eggertsson and Woodford (2003) that the optimal escape from a liquidity trap is for the central bank to commit to a higher future price level. However, it is also well understood that such commitment is not time-consistent and, therefore, not credible. Central banks have incentives to renege on such commitments. The model in this paper contributes to the literature by showing that the profit concerns introduced in the model can be used as a *commitment device* that incentivizes the central bank to commit to a higher future price level in a time-consistent manner.

I demonstrate this by assuming two types of central banks: The first central bank has full commitment power and doesn't have profit concerns. This central bank is assumed to optimize once and must follow through with its commitment for all subsequent periods. As a result, it could escape the liquidity trap due to the assumption of full commitment power. On the other hand, the other central bank is assumed to have no commitment power and with profit concerns (captured by the asymmetric objectives described above). This central bank optimizes every period and has a time-consistent policy. The model's main finding is that when the nominal interest rate is at the ZLB, the equilibrium allocations are numerically similar under the two types of central banks.

To provide intuition, note that depreciating the local currency leads to central bank profits under the model's setting, which is consistent with the empirical facts. Moreover, the model also provides a one-to-one relationship between inflation and depreciation (see section 5 for more details). As a result, profit concerns become a commitment device that allows the central bank to commit to a higher future price level *through future currency depreciation*. Committing to a higher future price level and a future depreciation is consistent with the central bank's goal of minimizing losses. Therefore, the commitment is time-consistent.

Data are simulated from the model to perform welfare analysis. The simulation assumes different probabilities for the economy to fall into a liquidity trap. The results indicate that when the ZLB constraint on the nominal interest rate is never binding (the economy is never in a liquidity trap), welfare monotonically decreases with central bank profit concerns. However, when the probability of falling into a liquidity trap increases, welfare *increases* for moderate profit concerns. This means profit concerns increase welfare when the economy has a chance of falling into a liquidity trap.

The intuition is straightforward. Under normal circumstances, profit concerns are simply a "distraction" for the central bank and cause inflation to be higher than optimal. However, when the economy falls into a liquidity trap occasionally, such concerns can help the economy escape a liquidity trap and increase welfare. As a result, how profit concerns affect welfare depends on how frequently the economy falls into a liquidity trap. As the probability of such an event increases, profit concerns become welfare-increasing.

My model demonstrates that when the nominal interest rate is at the ZLB, profit concerns create a commitment mechanism that allows central banks to raise future price levels in a time-consistent manner, which leads to an optimal escape from the liquidity trap. Simulation results further show that profit concerns decrease welfare under normal circumstances. However, they have the opposite effect when the probability of the economy falling into a liquidity trap increases.

The rest of the paper is organized as follows. Section 2 provides a literature review. Section 3 discusses the data source and related summary statistics. Section 4 demonstrates how the foreign exchange rate affects central banks' profit and provides evidence for central banks' intervention in the foreign exchange market for profit concerns. In section 5, a model is laid out to show how central bank profit concerns can serve as a commitment device and help the economy escape a liquidity trap. Section 6 concludes.

2 Literature Review

This paper contributes to four strands of literature. First, this study adds to a growing body of literature on the relationship between monetary policy and the central bank's concerns about profit and balance sheets. Del Negro and Sims (2015) and Benigno and Nisticò (2020) theoretically demonstrate that such concerns lead to higher inflation, while Berriel and Bhattarai (2009) show that the profit concerns lead to higher output gap variance. Empirically, Goncharov et al. (2023) and Klüh and Stella (2008) document the correlation between profit concerns and high inflation. Adler et al. (2016) shows that a central bank's weak financial conditions are correlated to a higher inflation rate in developing countries, while Pinter (2018) shows central banks produce higher inflation when fiscal support from the government is absent. On the other hand, Benecká et al. (2012) find no correlation between central banks' financial health and monetary policy. This study contributes to this literature by demonstrating that central banks worldwide intervene in foreign exchange markets for profit reasons. This research is the first to establish the causal effect of profit concerns on monetary policy.

Second, this research is linked to the vast literature on liquidity traps and the nominal interest rates' zero lower bound (ZLB). Sims et al. (2023), Del Negro et al. (2017), Gertler and Karadi (2013), and Gertler and Karadi (2011) have focused on the role of quantitative easing (QE) in a liquidity trap. They find that QE mitigates the effect of financial distress by decreasing credit spreads and minimizing credit market disruption. On the other hand, Eggertsson and Woodford (2003) and Krugman (1998) demonstrate that the optimal escape from a liquidity trap is for the central bank to commit to the ex-ante optimal policy. These commitments, however, may not be time-consistent. Jeanne and Svensson (2007) and Bhattarai et al. (2022) further show that central banks are concerned with their balance sheet and equity level, and such concerns have the potential to transform those commitments to become time-consistent. My search adds to Jeanne and Svensson (2007) by using a dynamic model and shows that a similar result can still be achieved when central banks are concerned with their flow profits instead of equity level. This paper also adds to Bhattarai et al. (2022) by assuming a more realistic central bank with an asymmetric preference toward profit. My assumption captures the fact that central banks avoid losses but do not maximize profits. This research contributes to this literature by showing that an independent central bank with asymmetric profit concerns, which is supported by empirical findings, can serve as a commitment device and help mitigate financial distress when ZLB is binding.

Third, this paper is closely related to the literature on central bank asymmetric foreign exchange intervention. The seminal study by Calvo and Reinhart (2002) points out that many emerging countries have a "fear of floating" between the 1970s and 1990s. Depreciation triggers fears of financial distress and/or inflation pass through. As a result, countries intervene aggressively when facing depreciation pressure but not appreciation pressure. Benlialper and Cömert (2016) also demonstrates this asymmetric fear of depreciation. However, much recent research shows that these asymmetric preferences have reversed. Many central banks now have a “fear of appreciation” (or “reversed fear of floating”) (see Keefe and Shadmani (2018), Chen (2016), Levy-Yeyati et al. (2013), Pontines and Siregar (2012), and Pontines and Rajan (2011), for example). Central banks now intervene aggressively when facing appreciation pressure, but not depreciation pressure, to insure against a potential currency crisis or to stimulate trade and growth. This study provides an alternative rationale for such asymmetric preference—central banks with profit concerns prefer depreciation over appreciation since it leads to central bank profits.

Finally, this research is related to the vast accounting literature on real earnings management. Companies often alter reported earnings by manipulating actual business activities rather than through accounting practices. This type of earnings management is intended to meet certain financial targets or expectations, often to influence perceptions of the company's performance by investors, analysts, or other stakeholders (see Thomas et al. (2022), Caskey and Ozel (2017), Roychowdhury (2006), Graham et al. (2005), and Erickson et al. (2004), for example). This research contributes to this literature by demonstrating that central banks worldwide are no exceptions and they also exhibit a pattern of real earnings management. In particular, they would use monetary policies to avoid reporting losses.

3 Data

One main focus of this research is how central bank profit concerns impact monetary policies. In particular, how profit concerns influence foreign exchange intervention (FXI). The FXI measurements are based on Adler et al. (2024). The traditional FXI is typically proxied by using the change in the stock of the central bank's foreign reserves, measured in USD. Adler et al. (2024) improves upon traditional proxies in several dimensions. First, it accounts for both spot and derivative transactions. Moreover, it adjusts for the foreign reserves' valuation changes and periodic dividend payments. These are not FXI since they do not entail buying or selling foreign currency but are often ignored by the traditional proxies. Lastly, the new FXI measurement also adjusts reserve changes for a broader range of operations with residents and non-residents. Consider a central bank that borrows from the IMF or accepts foreign currency deposits from commercial banks. These operations increase foreign reserves for the central bank but are not interventions. The traditional FXI proxy ignores these aspects, while the new estimate fully adjusts for these and other similar operations.

Central banks that belong to a currency union (e.g., The Banque de France) do not have complete control over exchange rate policies. Therefore, they are excluded from this paper. This research also excludes data on supranational central banks (e.g. European Central Bank) and local central bank branches. This yields a panel of 116 central banks from 2000 to 2024. The FXI observations are of monthly frequency, and not all central banks have data for all years. On average, each central bank has 266 months (22.2 years) of data. Another important aspect of the dataset is the fiscal year end for each central bank. Different central banks would report their profit in different calendar months. This research documents the last fiscal month for all the 116 central banks included in the dataset. The information is obtained manually from each central bank's financial statements or website. 21 out of the 116 central bank's fiscal years do not end in December, with June being the most common alternative. Moreover, 4 central banks change their fiscal year during the sample period. Table 1 provides an overview of the data on FXI. The table lists the 116 central banks included in this study, each central bank's first year in the dataset, the number of monthly FXI observations available, and the last fiscal month for each central bank.

Other data used in this research included the financial statements from central banks, exchange rates, foreign reserves, and central bank de jure independence. Central bank income statements are reported annually and collected from S&P Capital IQ Pro. The balance sheet, foreign reserves, and exchange rates data are of monthly frequencies and are collected from IMF's International Financial Statistics. Finally, Dincer and Eichengreen (2014) provides information on central bank de jure independence.

Table 1. Sample composition by country

Country	First year	Obs.	Last Month
Afghanistan	2009	147	03/12*
Albania	2000	289	12
Algeria	2000	285	03
Angola	2000	289	12
Argentina	2000	289	12
Armenia	2000	289	12
Australia	2000	289	06
Azerbaijan	2000	289	12
Bahamas	2000	287	12
Bahrain	2000	287	12
Bangladesh	2000	281	06
Belarus	2000	289	12
Bolivia	2000	283	12
Bosnia and Herzegovina	2002	265	12
Botswana	2000	287	12
Brazil	2000	289	12
Brunei Darussalam	2002	263	12
Bulgaria	2000	289	12
Cambodia	2000	283	12
Canada	2000	289	12
Chile	2000	289	12
China	2000	289	12
Colombia	2000	289	12
Costa Rica	2000	289	12
Croatia	2000	276	12
Cyprus	2000	96	12
Czech Republic	2000	289	12
Democratic Republic of the Congo	2000	272	12
Denmark	2000	289	12
Dominican Republic	2000	288	12
Ecuador	2000	289	12
Egypt	2000	288	06
El Salvador	2000	288	12
Estonia	2000	132	12
Ethiopia	2000	257	06
Georgia	2000	289	12
Ghana	2000	289	12
Greece	2000	12	12
Guatemala	2000	289	12
Guinea	2000	256	12
Guyana	2000	288	12
Honduras	2000	289	12
Hong Kong	2000	289	12
Hungary	2000	288	12
Iceland	2000	289	12
India	2000	289	03/06*
Indonesia	2000	289	12
Iraq	2000	278	12
Israel	2000	289	12
Jamaica	2000	288	12
Jordan	2000	289	12
Kazakhstan	2000	289	12
Kenya	2000	289	06
Korea	2000	289	12
Kuwait	2000	289	03
Latvia	2000	168	12
Lebanon	2000	276	12
Libya	2000	289	12
Lithuania	2000	180	12
Macao	2001	237	12
Madagascar	2000	288	12
Malaysia	2000	289	12
Malta	2000	93	12
Mauritius	2001	278	06
Mexico	2000	289	12
Moldova	2000	289	12
Mongolia	2000	275	12
Morocco	2001	275	12
Mozambique	2000	288	12
Myanmar	2001	244	09/03*
Namibia	2000	289	12
Nepal	2000	288	07
New Zealand	2000	289	06
Nicaragua	2000	289	12
Nigeria	2000	283	12
North Macedonia	2001	276	12
Norway	2000	289	12
Oman	2000	289	12
Pakistan	2000	286	06
Panama	2000	289	12
Papua New Guinea	2000	281	12
Paraguay	2000	289	12
Peru	2000	288	12
Philippines	2000	289	12
Poland	2000	289	12
Qatar	2000	289	12
Romania	2000	286	12
Russian Federation	2000	289	12
Rwanda	2000	289	06/12*
Saudi Arabia	2005	228	06
Serbia	2006	216	12
Singapore	2000	288	03
Slovak Republic	2000	108	12
Slovenia	2000	84	12
South Africa	2000	289	03
Sri Lanka	2000	276	12
Sudan	2000	216	12
Sweden	2000	289	12
Switzerland	2000	289	12
Taiwan	2000	289	12
Tanzania	2004	182	06
Thailand	2000	288	12
Trinidad and Tobago	2000	289	09
Tunisia	2000	289	12
Turkiye	2000	289	12
Uganda	2000	232	06
Ukraine	2000	289	12
United Arab Emirates	2000	289	12
Uruguay	2000	289	12
Uzbekistan	2000	134	12
Venezuela	2000	221	12
Vietnam	2001	275	12
West Bank and Gaza	2006	216	12
Yemen	2000	253	12
Zambia	2000	275	12
Zimbabwe	2000	276	12

"First year" indicates the country's first year included in the dataset. "Obs." is the number of monthly foreign exchange intervention observations obtained for each central bank. "Last Month" indicates the last fiscal month for each central bank, with * representing the central bank changed its last fiscal month during the sample period.

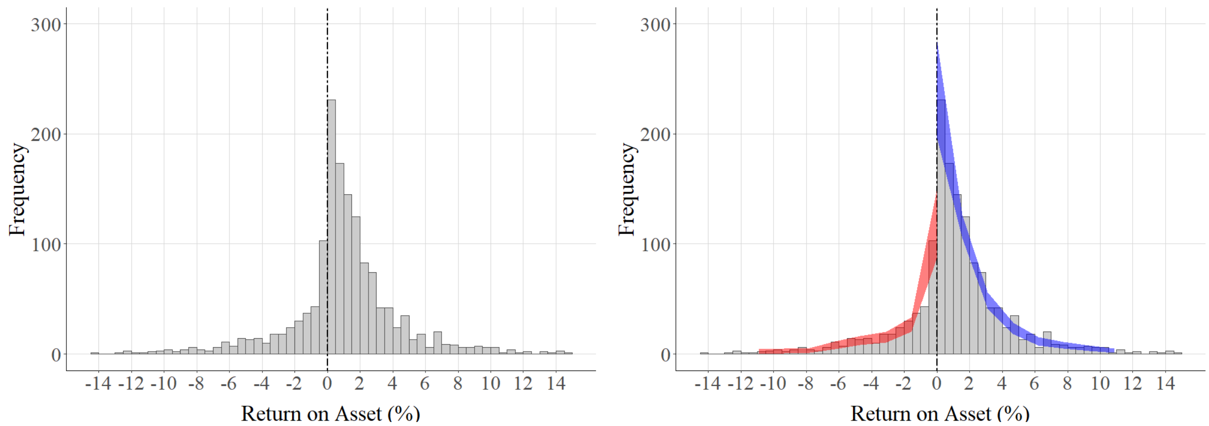
4 Empirical Results

This section investigates the relationships between central banks' policies and profit concerns. First, we look at the central banks' profit distribution and demonstrate evidence of profit management. Second, we focus on how local currency depreciation can lead to central banks' profit. Third, we establish the causal relationship between profit concerns and foreign exchange intervention (FXI). This section shows that interventions to increase profits are particularly likely right before central banks are about to release their financial statements, pointing to a causal effect of profit concerns on intervention. Furthermore, these interventions mainly occurred when central banks had strong incentives to avoid losses, strengthening the conclusion that these interventions are motivated by profit concerns.

4.1 Central Bank Profit Distribution

Of the 116 central banks in this study, 95 have income statements available. There are 1,496 bank-year observations (around 15.7 years of data per central bank) on annual net income. To compare central banks of different sizes, I measure central bank profit using the return on assets (RoA): The ratio of periodic net income over the beginning-of-the-period total asset. Figure 1 shows the histogram of central bank profits. The left panel is the histogram of central banks' RoA, which demonstrates that central banks are discontinuously more likely to report a small positive profit than a small negative profit. The right panel is the same histogram with the local polynomial density estimator proposed in Cattaneo et al. (2020), where the shaded area represents a 95% confidence interval. The confidence intervals do not overlap at RoA $= 0$, indicating that the jump at zero is statistically significant. Note that in a standard central banking model, profits are supposed to be entirely irrelevant. Any profit level, including zero, is not a fundamentally important number, and the central bank profit distribution should be smooth. Therefore, a discontinuity in the profit distribution at zero should be considered a sufficient condition for central bank profit concerns. The dis-

Figure 1. The Distribution of Central Banks' Return on Assets



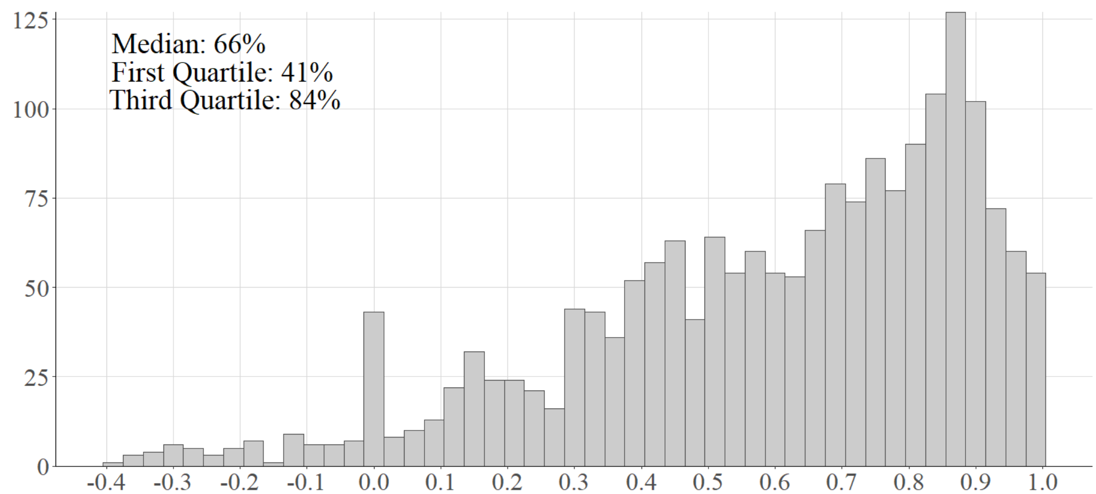
The left panel shows the histogram of central bank profits as measured by the return on assets (periodic net income over the beginning-of-the-period total asset). The right panel displays the same histogram with the local polynomial density estimator proposed in Cattaneo et al. (2020), where the shaded area represents a 95% confidence interval. *Data Sources:* S&P Capital IQ Pro.

continuous behavior is a clear sign of profit management and is first discussed in Goncharov et al. (2023). The results here are consistent with their findings. In this section, we explore the method used by central banks to achieve this "jump". Namely, we'll see evidence of how central banks use FXI to avoid reporting losses.

4.2 Exchange Rate's Impact On Central Banks' Profit

Central banks worldwide profit from local currency depreciation due directly to accounting identity. Note that all central banks prepare their financial statements in local currencies regardless of the accounting practices. Consider a central bank that holds foreign reserves (assets denominated in foreign currencies). For that central bank, local currency depreciation would naturally lead to a re-evaluation gain on the reserves. That is, simply from an accounting point of view, local currency depreciation would result in a capital gain on the foreign reserves held by the bank. Moreover, the larger the foreign reserves, the greater the gain from depreciation. This profit channel is crucial for central banks worldwide. Among the 116 central banks in this study, 93 of them have balance sheet data available. There are 1,906 bank-year observations (around 20.5 years of data per central bank) on foreign assets

Figure 2. Net Foreign Asset To Total Asset Ratio



This figure shows the histogram of the net foreign asset (foreign asset minus foreign debt) to total asset ratio for 93 central banks during the covered periods of 2000-2024. There are 1,906 bank-year observations, with the first quartile, median, and third quartile being 41%, 66%, and 84%, respectively. The ratio is upper bound by 1, and a negative value means net foreign debt. *Data Sources:* IMF's International Financial Statistics.

and total assets positions. Figure 2 plots the net foreign asset (foreign asset minus foreign debt) to total asset ratio for the 93 central banks from 2000-2024. The ratio is upper bound by 1, and a negative value means net foreign debt. The first quartile, median, and the third quartile are 41%, 66%, and 84%, respectively. Given the large amount of foreign reserves central banks generally hold, a small fluctuation in the exchange rate could hugely impact the profit a central bank reports. A small local currency depreciation could lead to large central banks' profit, and a small appreciation could lead to sizable losses.

4.3 Foreign Exchange Intervention: The Last Fiscal Month

This section first focuses on the FXI measurement used in this paper, then on the timing of earnings reports for central banks. Finally, it demonstrates that the FXI that puts depreciation pressure on the local currency is pervasive right before central banks release their financial statements, pointing to a causal effect of profit concerns on such interventions.

The FXI is commonly estimated from the change in the stock of the central bank's foreign reserves. This measurement is widely available but has several problems. First, the movements may be due to valuation changes. Moreover, the foreign reserves that the central bank owns yield dividends periodically. These dividends should be adjusted since these are not active interventions by the central bank. Furthermore, adjustments should also be made when the central bank has foreign currency transactions with residents and non-residents. Central banks borrow from and repay loans to foreign entities like the IMF; they also accept deposits or withdrawals of foreign currencies by the government or private sectors. Adler et al. (2024) adjusts for the factors mentioned above and provides a refined FXI dataset. The FXI measurement used in this research is based on on their dataset and is of monthly frequency. There are 30,191 FXI observations from 116 countries from 2000 to 2024, and the data composition is in Table 1. The FXI is measured in percentage points of 3-year moving average nominal GDP to compare across countries.

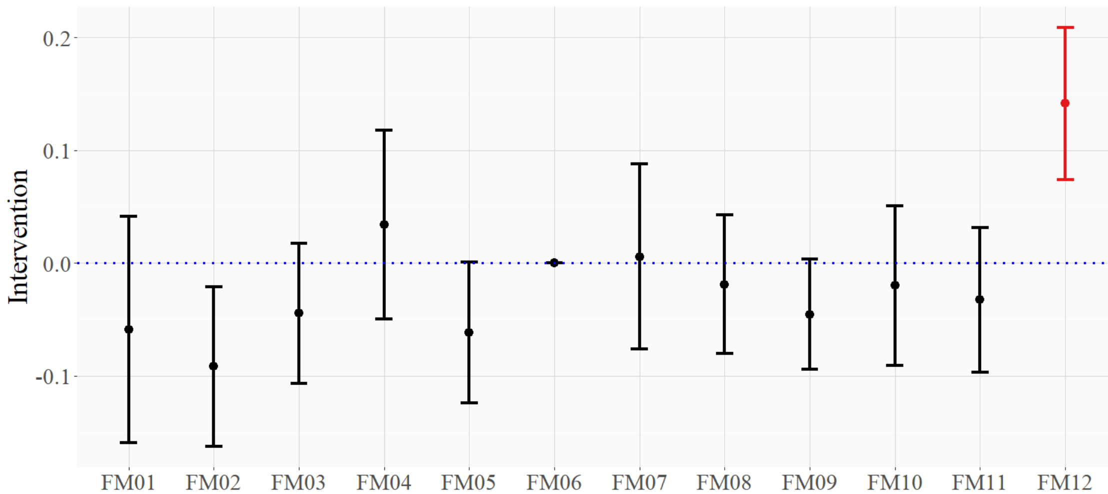
Regarding the time of profit reporting, central banks worldwide report their profit at the end of the fiscal year, which may *not* be December 31. The information on the fiscal year-end is hand-collected from central bank income statements and websites. In the dataset, 21 of the 116 central banks' fiscal years do not coincide with the calendar year (with the fiscal year ending in June being the most common alternative).

Having the measurement of FXI and the information about the time of report, we can see how central banks' FXI differ across each *fiscal* month by running the following regression:

$$
\begin{aligned}
Y_{i,t,m} = \alpha_{i,t} &+ \beta_1 \cdot \mathbb{I}(m = \text{first fiscal month for country } i) \\
&+ \beta_2 \cdot \mathbb{I}(m = \text{second fiscal month for country } i) + ... \\
&+ \beta_{12} \cdot \mathbb{I}(m = \text{last fiscal month for country } i) + \epsilon_{i,t,m},
\end{aligned}
\tag{1}
$$

where $Y_{i,t,m}$ is the FXI in percentage points of (3-year moving average) GDP for central bank i at the year t month m. $\alpha_{i,t}$ is the country-year fixed effect and $\mathbb{I}(\cdot)$ is the indicator function. β_6 (the intervention in the middle of the fiscal year) is left out as a baseline. Note that by increasing $Y_{i,t,m}$, central bank i actively buys foreign currencies and releases local

Figure 3. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year



This figure shows the estimation results for equation (1). 95% confidence intervals are displayed, and standard errors are clustered for central banks. The key parameter of interest β_{12} is significant with the point estimate of 0.14. *Data Sources:* S&P IQ Pro and Adler et al. (2024)

currencies, which creates depreciation pressure on the local currency at year t, month m. $Y_{i,t,m}$ is trimmed at the 1^{st} and 99^{th} percentiles to control for outliers. β_j, for $j \neq 6$ captures the difference of central bank FXI between the fiscal month j and the middle of the fiscal year. We are particularly interested in β_{12}, the behavior in the last fiscal month when central banks were just about to release financial statements. Note that without profit concerns, the last fiscal month should *not* be different from any other month. Figure 3 display the estimations of equation (1). The intervals represent a 95% confidence ban, and the standard errors are clustered for central banks. Point estimates and standard errors are reported in Table 2. The point estimate for β_{12} is 0.141 and statistically significant[1]. This shows that central banks intervened aggressively in the last fiscal month, captured by the positive and significant β_{12}. This positive intervention would cause depreciation pressure on the local currency and help the central bank to report a higher profit.

An important critique of this finding would be a positive and significant β_{12} may be due

[1] The unconditional standard deviation for $Y_{i,t,m}$ is 0.745. Therefore, 0.14 is about 19% of the unconditional standard deviations.

Table 2. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year

	β_1	β_2	β_3	β_4	β_5	β_6
Estimate	−0.058	−0.091**	−0.044	0.034	−0.061*	(baseline)
Cluster standard Error	0.051	0.036	0.031	0.042	0.031	(baseline)
	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}
Estimate	0.005	−0.018	−0.045*	−0.020	−0.032	**0.141***
Cluster standard Error	0.041	0.031	0.024	0.036	0.032	**0.034**
Observations			30,191			
Country × year fixed effects			Yes			
Adjusted R^2			0.104			

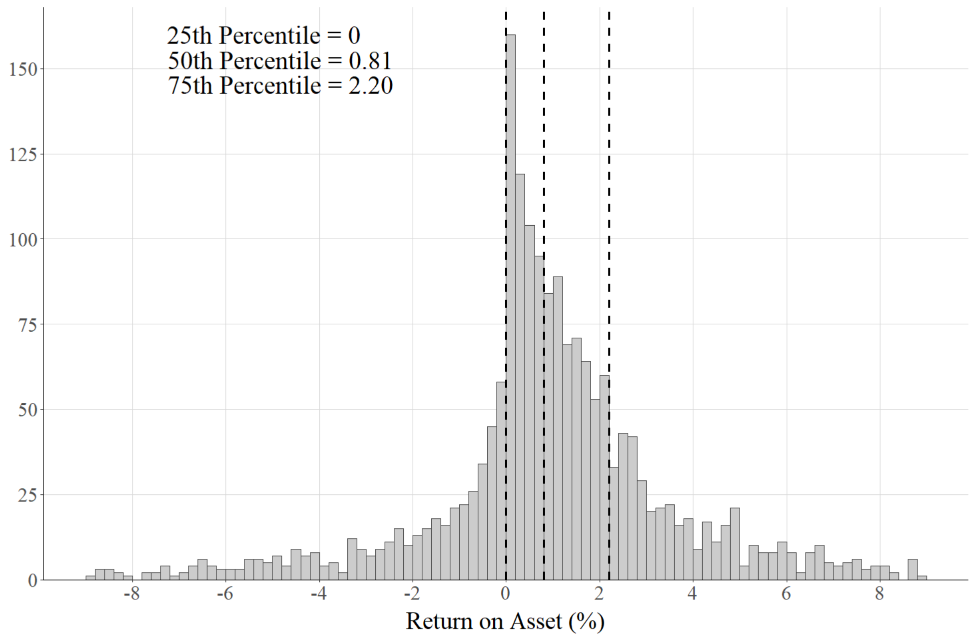
This table shows the estimation results for equation (1). Standard errors are clustered by central banks. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively. *Data Sources: Data Sources:* S&P IQ Pro and Adler et al. (2024).

to seasonality or other profit-unrelated reasons. The following sections address this critique by showing that the significance and magnitude of β_{12} varies predictably with central banks' *incentives* to avoid losses.

4.3.1 Previous Year Profits

In the last section, we see that interventions that increase profits are particularly likely right before central banks release their financial statements, pointing to a causal effect of profit concerns on intervention. Here, we show that this intervention pattern is most prevalent for central banks under financial pressure. It is reasonable to assume that central banks reporting a loss (or a lower profit) in the previous year are under more financial and political pressure than the ones reporting large profits. Since we can observe each central bank's previous year's profits, we can see how the intervention pattern differs for central banks reporting different profits. To start, I divide the data into four equal quartiles according to the profit reported at the end of the last fiscal year: previous year losses (RoA < 0), previous year small profits (RoA $\in [0, 0.81\%)$), previous year medium profits (RoA $\in [0.81\%, 2.20\%)$), and previous year large profits (RoA $> 2.20\%$). Figure 4 displays the RoA distribution for

Figure 4. The Distribution of Central Banks' Return on Assets: Four Quartiles

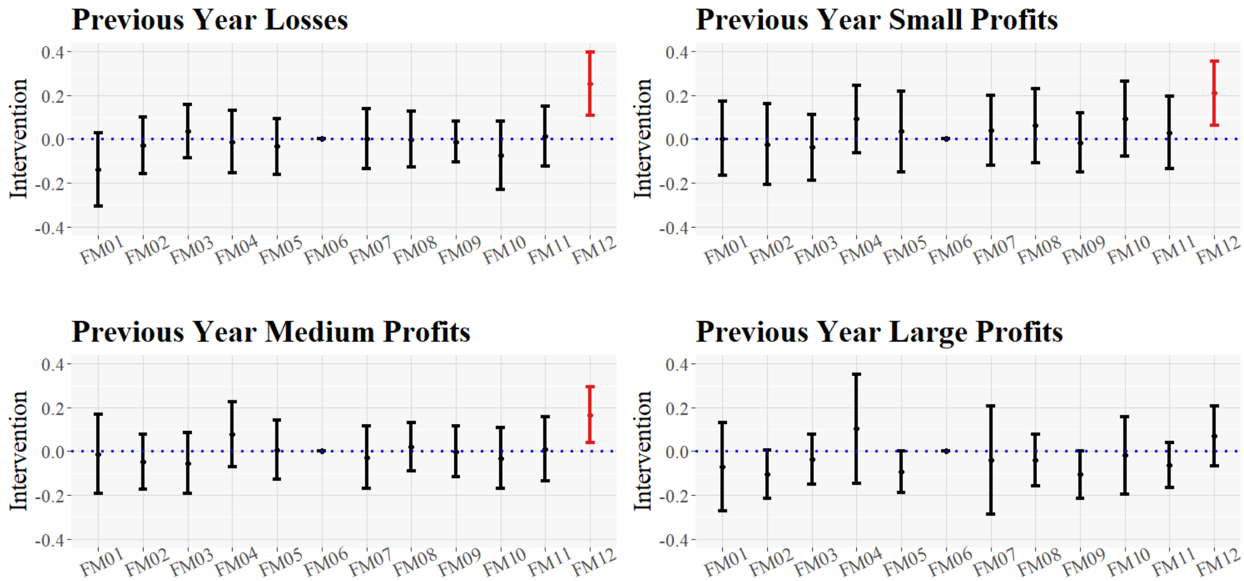


This figure displays the RoA distribution for central banks in the dataset. The data is divided into four equal quartiles according to the profit reported at the end of the last fiscal year: previous year losses (RoA < 0), previous year small profits (RoA $\in [0, 0.81\%)$), previous year medium profits (RoA $\in [0.81\%, 2.20\%)$), and previous year large profits (RoA $> 2.20\%$). *Data Sources:* S&P Capital IQ Pro.

the central banks and the corresponding four quartiles.

We then test equation (1) using different subsamples; the results are given in Figure 5. The top-left panel is the regression results (estimation of equation (1)) using the subsample for central banks reporting a loss in the last fiscal year. The point estimate of β_{12}, our key variable of interest, is 0.25 and significant at the 1% level. The magnitude nearly doubled compared to the estimate using the entire sample (0.14). In comparison, the top-right and lower-left panel are the regression results using the subsample for central banks that report small profits (RoA $\in [0, 0.81\%)$) and medium profits (RoA $\in [0.81\%, 2.20\%)$) in the previous year. The point estimates of β_{12} are 0.21 (significant at the 1% level) and 0.16 (significant at the 5% level), respectively. Finally, the lower-right panel shows the results for central banks that reported a large profit (RoA $> 2.20\%$) in the last fiscal year. The point estimate is 0.07 and is not statistically significant. Here, we see that the central bank that reported a loss in the previous year will intervene aggressively in the last fiscal month of the following

Figure 5. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year (Four subsamples according to the previous year profits)



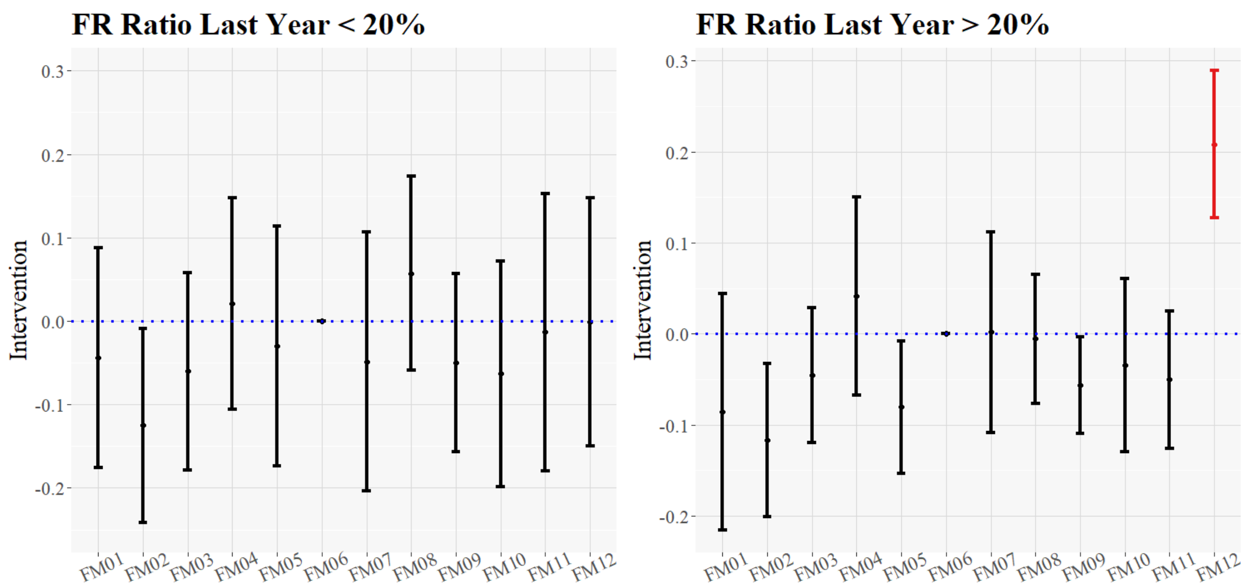
This figure shows the estimation results for equation (1) for four sub-samples. The data is divided into four groups according to the profit reported at the end of the fiscal year: previous year losses (RoA < 0), previous year small profits (RoA $\in [0, 0.81)$), previous year medium profits (RoA $\in [0.81, 2.20\%)$), and previous year large profits (RoA > 2.20). The bar represents a 95% confidence interval. Standard errors are clustered for central banks. *Data Sources:* S&P Capital IQ Pro and Adler et al. (2024).

year. The same pattern is still present, although it decreases in magnitude, for central banks that report a small to medium profit. Critically, the intervention pattern in the last fiscal month is not present for central banks that reported a large profit last year. Note that if the distortion pattern is not due to profit concerns, we should observe this behavior no matter what profit central banks report. Instead, we observe this behavior for all the central banks *except* for the ones that reported large profits last year. This result is also intuitive; for central banks already in a financially comfortable situation, there's less incentive to intervene in the foreign exchange market to increase profits. The results in Figure 5 show that central banks with more incentives intervene more aggressively and provide further evidence that the intervention is due to profit concerns.

4.3.2 Foreign Asset Ratio

In previous sections, we have discussed how the exchange rate impacts the central bank's profit. Recall that from an accounting point of view, local currency depreciation would result in a capital gain on the foreign reserves held by central banks. Moreover, the larger the foreign reserves, the greater the gain from depreciation. Figure 2 plots the net foreign asset to total asset ratio for the 93 central banks in our dataset that have balance sheet data. The first quartile, median, and third quartile are 41%, 66%, and 84%, respectively. Given the large amount of foreign reserves central banks generally hold, a small fluctuation in the exchange rate could hugely impact the profit a central bank reports. However, there are some central banks in certain periods that hold a small amount of foreign reserves. For those central banks, the foreign exchange rate would have little impact on their profits. As a result, even with profit concerns, central banks with small foreign reserves would have little incentive to intervene in the foreign exchange market for profit motives. This is exactly what we observed in the data. Figure 6 displays the results. The data are grouped into two sub-samples based on the net foreign asset to total asset ratio at the beginning of the fiscal year. A central bank in a given year with a ratio less than 20% is separated from those greater than 20%. The former group accounts for 12.4% of the total available data on FXI (2,685 observations), while the latter group accounts for the remaining 87.6% (18,954 observations). The left panel of Figure 6 displays the regression result from equation (1) using the sub-sample with a low ratio. It shows how the central bank intervenes each fiscal month, given that it has a small foreign reserve at the beginning of the fiscal year. The point estimate of the key variable of interest, β_{12}, is 0. On the other hand, the right panel shows the same estimation using the sub-sample with a high net foreign asset to total asset ratio. The point estimation for β_{12} is 0.208 and statistically significant at a 1% confidence level with standard error clustered for central banks. The results demonstrate that central banks with large reserves tend to intervene aggressively at the fiscal year-end. On the other hand, the same pattern cannot be observed for central banks that start the fiscal year with a small

Figure 6. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year (Two subsamples according to the net foreign asset to total asset ratio)



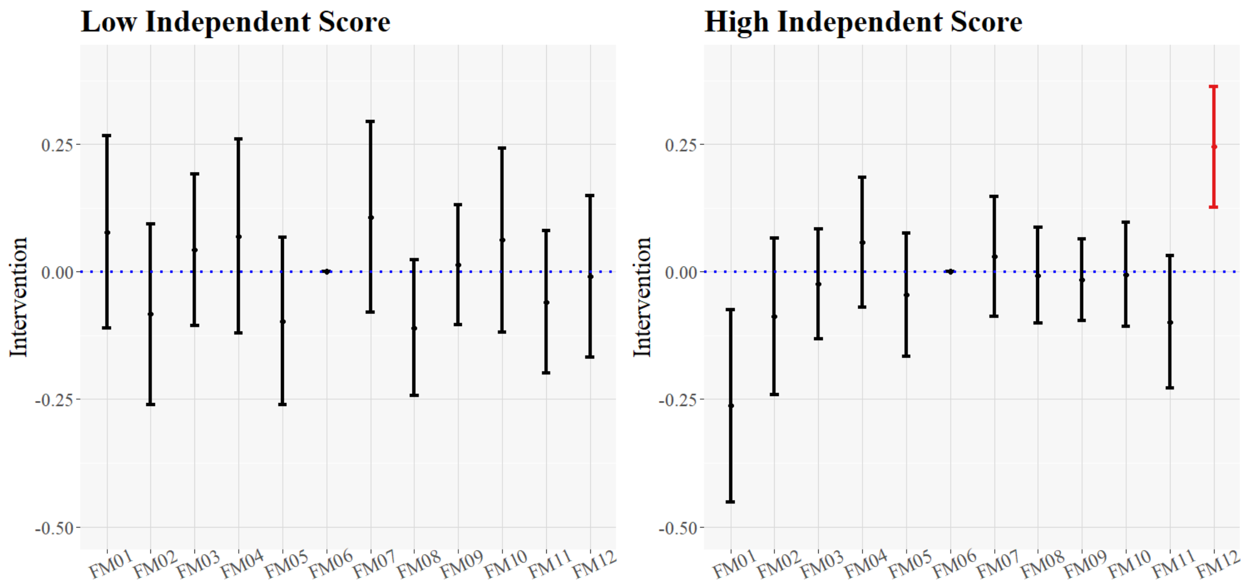
This figure shows the estimation results for equation (1) for two sub-samples. The data is divided into two groups according to the net foreign asset (foreign asset minus foreign debt) to total asset ratio: the central bank-year with a ratio less than 20% (left panel) and more than 20% (right panel). These two groups account for 12.4% and 87.6% of the available data on FXI (2,685 and 18,954 observations), respectively. The bar represents a 95% confidence interval. Standard errors are clustered for central banks. *Data Sources:* IMF's International Financial Statistics and Adler et al. (2024).

foreign reserve on their balance sheet. The results are intuitive as the latter group of central banks has fewer incentives to intervene for profit motives than the former group.

4.3.3 Central Bank de jure Independence

One key reason central banks care about the profits they report is to protect their independence. The quotes at the beginning of this paper from the Federal Reserve and the Riksbank are anecdotal evidence. Central bank losses may be viewed by the government or the public as a sign of incompetence and politicized at the expense of the independence of the central bank. Goncharov et al. (2023) provide empirical evidence on this matter. They show that central banks are discontinuously more likely to report slightly positive profits than slightly negative profits in general, but legally independent central banks exhibit a

Figure 7. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year (Two subsamples according to the independence index)



This figure shows the estimation results for equation (1) for two sub-samples. The data is divided into two equal groups according to the independence index provided by Dincer and Eichengreen (2014). Each group contains approximately 5,900 observations. The bar represents a 95% confidence interval. Standard errors are clustered for central banks. *Data Sources:* Dincer and Eichengreen (2014) and Adler et al. (2024).

larger discontinuity. This result is consistent with the assumption that independent central banks may have stronger incentives to avoid losses.

This research shows that legally independent central banks tend to intervene in the foreign exchange market for profit concerns. At the same time, such a pattern is *not observed in less independent central banks*. This finding confirms the hypothesis that independent central banks may have stronger incentives to avoid losses. The measurement of central bank independence is based on Dincer and Eichengreen (2014). The authors pose 24 questions covering different aspects of central bank legal independence, including policy choice, objectives, and governance structures. They assign scores to central banks from 1994 to 2014, which range from zero to one, with higher values indicating more independent central banks. Among the 116 central banks covered by this study, 70 have data on the independence index from 2000 to 2014.

Figure 7 displays the results. The data are grouped into two equal groups according to the independence index provided by Dincer and Eichengreen (2014). Each group contains approximately 5,900 FXI observations. The left panel of Figure 7 displays the regression result from equation (1) using the sub-sample with central banks that received low independent scores. These are the central banks that are more integrated with the government. The point estimate of the key variable of interest, β_{12}, is -0.009 and is not statistically significant. On the other hand, the right panel shows the same estimation using the sub-sample with high independent scores. The point estimation for β_{12} is 0.245 and statistically significant at a 1% confidence level with standard error clustered for central banks. The results demonstrate that independent central banks are more likely to intervene for profit reasons than their less independent counterparts. This is consistent with the assumption that independent central banks avoid losses to protect their independence.

In the empirical section of this research, I analyze data from 116 central banks between 2000 and 2024. The findings reveal that central banks are disproportionately more likely to report a small positive profit than a small negative one, indicating a strong concern for profit. I also show that local currency depreciation boosts profits due to the revaluation gains on foreign reserves held by central banks. The main takeaway is that central banks frequently intervene in the foreign exchange market in the last fiscal month before releasing financial statements, causing depreciation and increasing profits. These interventions are driven by profit motives, especially in central banks that reported losses in the previous year, has large reserves, and those with higher independence. The distinct distortion pattern observed supports the idea that profit concerns motivate these interventions rather than other factors, such as seasonality. In the next section, I explore how these profit concerns can act as a commitment device, potentially helping economies escape liquidity traps.

5 Profit Concerns and Liquidity Trap: A Simple Model

The model used in this paper builds on a standard New Keynesian model. Time is infinite and discrete. Representative households and a continuum of monopolistically competitive firms produce differentiated goods and set prices under the Calvo (1983) setting. Demand shocks from the representative households are the only source of uncertainty in the model. Wages are flexible, and capital accumulations are ignored. The model is one of a small open economy with access to complete international financial markets, and the law of one price holds. Beyond the standard setup, two important additions are added to the model: (i) Zero lower bound on the nominal interest rate is introduced, and (ii) there's an independent central bank with periodic accounting profits and asymmetric profit concerns.

Since the model is relatively standard, the sections below only lay out the main assumptions while relegating most derivations to the Appendix. Subsection 5.1 describes the basics of the model. 5.2 then demonstrate how profit concerns can be viewed as a commitment device and help the economy escape a liquidity trap. 5.3 performs welfare analysis on central bank profit concerns.

5.1 Model

In this section, I outline the problems households and firms face, the property of the small open economy, the central bank's accounting profit, equilibrium conditions, and calibration of the model parameters. Apart from the settings regarding the central bank, which is a new feature of the model, the other model settings follow the framework developed in Galí and Monacelli (2016) and Galí (2015) closely.

5.1.1 Households

The small open economy is inhabited by representative households seeking to maximize:

$$\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \\
=&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}\right] Z_t,
\end{aligned}$$

where C_t is the composite consumption index defined by

$$C_t \equiv \left(\frac{C_{H,t}}{1-\nu}\right)^{1-\nu} \left(\frac{C_{F,t}}{\nu}\right)^{\nu},$$

with $\nu \in [0, 1]$ is (inversely) related to the degree of home bias and a natural measurement of openness for the home economy. $C_{H,t}$ is the domestic good consumption index defined by

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},$$

where $i \in [0, 1]$ is the good variety produced by domestic firms that will be discussed below, and $\epsilon > 1$ denotes the elasticity of substitution among the varieties of domestic goods. $C_{F,t}$ is the quantity of imported goods consumed. N_t is the employment (or working hours) of the household, and φ determines the curvature of the disutility of labor.

Z_t is an exogenous preference shifter, where its log, $z_t \equiv \log Z_t$, follows an exogenous AR(1) process $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ with ε_t^z being white noise. Note that shocks to z_t affect the marginal rate of substitution among goods at different times, which in turn will change the demand for consumption. Therefore, shocks to z_t is henceforth referred to as *demand shocks*. Moreover, the demand shock z_t is the only exogenous shock in the model.

Define the *consumer price index* (CPI) as $P_t \equiv (P_{H,t})^{1-\nu}(P_{F,t})^{\nu}$. $P_{H,t}$ is an aggregate price index for domestic consumption and is given by $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$. $P_{H,t}(i)$ is the price of $C_{H,t}(i)$ and $P_{F,t}$ is the price of imported goods $C_{F,t}$ measured in *domestic* *currency.* The period budget constraint for the representative household is expressed in domestic currency and given by

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + P_{F,t}C_{F,t} + \mathbb{E}_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_tN_t - T_t + \Lambda_t, \tag{2}$$

where D_{t+1} is the nominal payoff in period $t+1$ of the portfolio purchased in period t and $Q_{t,t+1} \equiv \beta(C_t/C_{t+1})(P_t/P_{t+1})$ is the one-period ahead stochastic discount factor. W_t is the nominal Wage. T_t is the lump-sum tax (if positive) or transfer (if negative). Λ_t is the dividends from owning the firms.

The model assumes that the law of one price holds for import and export goods at all times. In particular:

$$P_{F,t} = \mathcal{E}_tP_t^*$$

where $\mathcal{E}_t$ is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, and P_t^* is the price of foreign goods *expressed in foreign currency.* P_t^* can also be interpreted as the world price index since $P_{H,t}$ does not affect the world price by the small economy assumption. Hereafter, P_t^* is assumed to be constant and normalized to unity. That is, $P_t^* = 1$ and $P_{F,t} = \mathcal{E}_t$ for all t.

5.1.2 Firms

Firm i in the home economy employs $N_t(i)$ of working hours and produces a differentiate good $Y_t(i)$ with a linear production function:

$$Y_t(i) = N_t(i). \tag{3}$$

Each firm may reset its price only with probability $1-\theta$, $\theta \in [0, 1)$, in any given period, independent of the past and each other. Prices are set in domestic currency and are the same for domestic and export markets. Following convention, the model assumes that the average markup is large enough and the shocks are small enough such that all firms meet the demand for their goods at the price they post. All firms also face identical nominal wage W_t when hiring labor. Employment is subject to a proportional (fixed) payroll tax τ (or subsidy if $\tau < 0$). Therefore, firms' common effective labor cost is $(1+\tau)W_t$.

5.1.3 International Risk-Sharing and Export

As in Galí (2015) and Galí and Monacelli (2016), this model assumes that households have access to a complete set of state-contingent securities, which leads to a standard international risk-sharing condition. As demonstrated in Appendix 7.2, the complete market assumption leads to the following simple relationship linking domestic consumption with (per capita) world consumption C_t^* and the terms of trade $\mathcal{S}_t \equiv P_{F,t}/P_{H,t}$ in the following way

$$C_t = C_t^* \mathcal{S}_t^{1-\nu} Z_t.$$

As in many standard New Keynesian model settings (see Galí and Monacelli (2016), Galí (2015), and Galí and Monacelli (2005) for example), here I assume the demand for exports of domestic good $i \in [0,1]$ is given by:

$$X_t(i) = \left(\frac{p_{H,t}(i)}{p_{H,t}}\right)^{-\epsilon} X_t,$$

where X_t is an index of aggregate export and assumed to be $X_t = \nu \mathcal{S}_t Y_t^*$.[2] In equilibrium, the world output equals world consumption, $Y_t^* = C_t^*$. This research considers a symmetric steady state where $\mathcal{S} = 1$. Henceforth, I denote a variable without the sub-script t as the steady state value of the original variable. We therefore have $X = \nu Y^* = \nu C^*$ and $C = C^*$ (by international risk sharing). Furthermore, $C_F = \nu C$ and $C_F = X$. That is, the trade is balanced in the steady state. Finally, the world's output and consumption are assumed to

[2]Both the demand schedule for exports of domestic good $i \in [0,1]$ and the aggregate export given in the paper can be micro-founded from the optimality conditions stemming from international households' decision. See Galí and Monacelli (2005) for more details.

be fixed for all t.

5.1.4 Central Bank's Accounting Flow Profit

For simplicity, the model assumes the only asset on the central bank's balance sheet is the non-interest-bearing foreign reserves F_t^*, which is expressed in foreign currency. To further simplify the model, I assume that the foreign reserves are fixed at a predetermined level F^*. That is, $F_t^* = F^*$ for all t.[3] As a result, the central bank's accounting flow profit each period is simply determined by the foreign reserve re-valuation gain/loss and is given by:

$$\Gamma_t = (\mathcal{E}_t - \mathcal{E}_{t-1})F^*.$$

The accounting flow profits are positive when local currency depreciates and negative when it appreciates. This is consistent with the empirical facts discussed in previous sections. Note that the accounting flow profits are, as the name suggests, purely accounting, and the central bank does not have any real income flows. Therefore, the central bank does not transfer resources to the household and vice versa.

To avoid negative values when taking logs, define $G_t = (Y + \Gamma_t)/Y$, which models central bank flow accounting profit as a fraction of steady-state output. Let $g_t = \log(G_t)$ and note that $\Gamma_t \geq 0$ if and only if $g_t \geq 0$. Log-linearize would yield:

$$g_t = F^* \Delta e_t, \tag{4}$$

where $\Delta e_t \equiv \ln(\mathcal{E}_t/\mathcal{E}_{t-1})$ is the percentage change in exchange rates, and $\Delta e_t > 0$ represents local currency depreciation. The model settings for the central bank are simplified and allow

[3] Alternatively, we can assume the central bank chooses F_t^* for each period. The exchange rate is subsequently determined by: $\Delta e_t = \beta_0 + \beta_1 \Delta f_t^* + \epsilon_t$, where $\Delta e_t \equiv \ln(\mathcal{E}_t/\mathcal{E}_{t-1})$, $\Delta f_t^* \equiv \ln(F_t^*/F_{t-1}^*)$, and $\beta_1 > 0$ captures the relationship between intervention and exchange rates. The simplified assumption is preferred since the foreign reserves F_t^* and the central bank's balance sheet are not the focus of this research. Moreover, the alternative assumption will not change the model's main results, which will be discussed in more detail in later sections.

us to see the mechanism transparently. Note that the main results in the later sections are robust to more complicated assumptions on the central bank's balance sheet. As long as depreciation leads to profit, the main results will hold. This will be discussed in more detail in Section 5.2.4. The objectives of the central bank will be laid out in sections 5.2.1.

5.1.5 Equilibrium

In Appendix 7.2, I derive the optimal conditions for the household and the firm's problem. I also derive the market clearing conditions and properties of the small open economy and log-linearize these equations around a zero-inflation steady state. A lowercase variable denotes the original variable's log deviation from the steady state (e.g., $c_t \equiv \log(C_t/C)$). After reorganizing, the equilibrium is condensed into the following system of difference equations :

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - (i_t - \mathbb{E}_t[\pi_{H,t+1}] - r_t^n) \tag{5}$$

$$\pi_{H,t} = \beta\mathbb{E}_t[\pi_{H,t+1}] + \lambda(1+\varphi)\tilde{y}_t \tag{6}$$

$$\Delta e_t = \tilde{y}_t - \tilde{y}_{t-1} + \pi_{H,t} - \phi(z_t - z_{t-1}) \tag{7}$$

$$g_t = F^*\Delta e_t \tag{8}$$

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} \tag{9}$$

$$z_t = \rho_z z_{t-1} + \epsilon_t^z, \tag{10}$$

where $\phi \equiv \frac{1+\varphi(1-\nu)}{1+\varphi}$, $\lambda = (1-\theta)(1-\beta\theta)/\theta$, and $\tilde{y}_t \equiv y_t - y_t^n$ denotes the output gap, with y_t^n being the natural level of output and r_t^n is the natural interest rate. As derived in the Appendix, both y_t^n and r_t^n are functions of the demand shock z_t and are given by:

$$y_t^n = -\frac{\nu}{1+\varphi}z_t, \quad r_t^n = (1-\rho_z)\phi z_t.$$

Output gap evolves according to the Euler equation (5), derived from the household's optimality conditions. Inflation evolves according to the New-Keynesian Phillips curve (6), derived from the firms' optimality conditions. Equation (7) expresses the depreciation rate as a function of output and inflation. This equation is derived by combining the law of one price, the international risk-sharing condition, and the goods market clearing condition. Equation (8) is the accounting flow profit for the central bank, Equation (9) is the definition for domestic inflation, and Equation (10) is the exogenous demand shock.

The equilibrium system characterizes the evolution of seven variables ($\tilde{y}_t, i_t, \pi_{H,t}, \Delta e_t, g_t, p_{H,t}$ and z_t) with six equations. We still need one equation to close the model. The final equation will be derived from the central bank's problem described in section 5.2.1, which would help us close and solve the model.

5.1.6 Calibration

Table 3 lists the settings for the model parameters. The parameters are well-parameterized in the literature. Most of the parameters in this paper follow Galí and Monacelli (2016) and Galí (2015).

Each period in the model corresponds to a quarter, and $\beta = 0.99$ as is common practice in the business cycle literature (this implies a steady state real interest rate of 4%). $\varphi = 5$ implies a Frisch elasticity of labor supply of 0.2. The elasticity of substitution among domestic goods varieties $\epsilon = 9$ implies a 12.5% of steady state average markup. θ, the Calvo index of price stickiness, is set to be 0.75. This means an average price duration of four quarters, a value consistent with empirical evidence.[4] The openness parameter, ν, is set to equal to 0.3, which is also the steady state import share of the economy. This is consistent with the average export and import share in the Eurozone countries. λ_1 represents the weight the central bank puts on the output gap relative to inflation in its objection function (to be discussed in section 5.2). It is set to be 0.05 as it is consistent with the second-order approximation of the welfare losses experienced by the representative household.[5] Finally, the foreign reserve held by the central bank F^* is set to be 0.45. The steady-state GDP of

[4] See, for example, Galí et al. (2001) and Galí et al. (2003).
[5] See Galí (2015), Appendix 5.1 for more detail.

Table 3. Calibration for the Model Parameters

Parameters	Descriptions	Values
φ	Curvature of labor dis-utility	5
ϵ	Elasticity of substitution among goods	9
θ	Calvo index of price rigidity	0.75
ν	Degree of openness	0.3
β	Discount factor	0.99
λ_1	Weight on output gap	0.05
F^*	Foreign Assets	0.45

the economy is normalized to be one. Therefore, this implies that the central bank's asset is 45% of the GDP, consistent with the empirical fact calculated by the author using balance sheet data from 116 central banks in 2021.

5.2 Profit Concerns As a Commitment Device

In this section, I show that profit concerns can be viewed as a commitment device that helps the economy escape the liquidity trap. Krugman (1998) argue that the optimal escape from a liquidity trap is for the central bank to commit to a higher future price level. However, he also pointed out that such commitment is not time-consistent and, therefore, not credible. To show that the central bank's profit concerns can serve as a commitment device, I first construct a central bank without profit concerns and under *commitment* - The central bank is assumed to have full credibility and can commit to any future policy (including non-time-consistent ones). As a result, the central bank under commitment can escape the liquidity trap. I then construct a *discretionary* central bank that *have* profit concerns and cannot commit to any future action. In each period, the central bank makes the optimal decision at the time (the action is always time-consistent). I show that in a liquidity trap, the equilibrium allocation for the discretionary central bank with profit concerns is *numerically similar* to the central bank under commitment.

The following subsections first outline the objectives of the two types of central banks and their respective optimal policies. Then, I show how a negative demand shock causes the economy to fall into a liquidity trap. Finally, I lay out the solution methods and demonstrate how profit concerns lead to the optimal escape from the liquidity trap.

5.2.1 Central Bank's Objective

In this paper, I analyze the equilibrium of the small open economy under two types of central banks. Under the first, which I refer to as the *commitment central bank*, the bank is assumed to be able to commit, with full credibility, to a policy plan. Under the second central bank, which I refer to as the *discretionary central bank*, the bank cannot commit to any future actions. The central bank re-optimizes each period without being bound by any earlier promises.

The commitment central bank is assumed to choose a state-contingent sequence $\{y_t, \pi_{H,t}\}_{t=0}^{\infty}$ that minimize:[6]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2\right]$$

$$s.t. \quad (6), \text{ and } i_t \geq 0.$$

The central bank's problem is solved in Appendix 7.3, and the following proposition is derived.

Proposition 1: the optimality condition for the commitment central bank when the ZLB is not binding ($i_t > 0$) is given by:

$$p_{H,t} = -\frac{\lambda_1}{\kappa_1}\tilde{y}_t, \tag{11}$$

where $\kappa_1 \equiv \lambda(1+\varphi) > 0$.

Literature has referred to this type of policy as a "targeting rule". The central bank under commitment seeks to maintain condition (11) between the target variables. The interpretation of this condition is straightforward: When facing inflationary pressure, the

[6] To simplify the calculation, the model assumes the central bank can choose the desired level of inflation and output gap each period. Alternatively, we can assume the central bank chooses the appropriate depreciation rate Δe_t to guarantee the desired outcome is achieved. Both assumptions would yield the same results. See Galí (2015) for more details.

central bank must drive the output below its natural level to dampen the rise of inflation. The central bank would keep pushing this "leaning against the wind" policy until the condition (11) is met. Note that the optimal condition requires the central bank under commitment to target *price level* instead of inflation. This will differ from the discretionary central bank's policy, which we discuss below.

The second type of central bank considered by this research is the discretionary central bank. The bank cannot commit to any future actions and is assumed to make whatever decision is optimal at the time. Therefore, we have a sequential optimization problem. Each period the central bank chooses $\{y_t, \pi_{H,t}\}$ to minimize

$$\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2 + \lambda_2 g_t^2 \cdot \mathbb{I}(g_t < 0)$$
$$s.t. \quad (6) \sim (8), \text{ and } i_t \geq 0,$$

where $\lambda_2 > 0$ is the central bank's profit concern and $\mathbb{I}(\cdot)$ is the indicator function. Note that when the flow profit g_t is positive, the third term in the objective function drops out. On the other hand, when $g_t < 0$, the central bank would minimize domestic inflation, output gap, and losses. This captures the central bank's asymmetric profit concern in the sense that it is not a profit maximizer. However, it does try to avoid losses. The discretionary central bank's problem is solved in Appendix 7.3, and the following proposition is derived.

Proposition 2: the optimality condition for the discretionary central bank when the ZLB is not binding ($i_t > 0$) is given by:

$$\begin{aligned} \pi_{H,t} &= -\frac{\lambda_1}{\kappa_1}\tilde{y}_t \qquad \text{if } g_t \geq 0 \\ \pi_{H,t} &= -\frac{\lambda_1}{\kappa_1}\tilde{y}_t - \lambda_2 \frac{\kappa_2}{\kappa_1} g_t \qquad \text{if } g_t < 0, \end{aligned} \tag{12}$$

where $\kappa_2 = F^*(1 + \kappa_1) > 0$.

Note that when the central bank's flow profit is positive, the optimal discretionary policy again requires the central bank to "lean against the wind". The central bank should drive up (down) inflation when there's a negative (positive) output gap. Despite the similarity, there is an important difference between the optimal policies of the two types of central banks. Although both central banks "lean against the wind", the central bank under commitment target *price level* while the discretionary central bank targets *inflation*. This has important implications for the equilibrium outcome and is well-documented in Galí (2015).

When the central bank's flow profit is negative, the discretionary central bank will still pursue the same policy, but now with an *inflationary bias*. Note that since $g_t < 0$, we have $-\lambda_2 \frac{\kappa_2}{\kappa_1} g_t > 0$. This implies that for any given level of inflation, a more expansionary policy will be taken by the discretionary central bank with profit concerns. The intensity of the inflation bias depends on the level of profit concern λ_2.

A central bank concerned about profit would pursue a more expansionary monetary policy due to its positive effect on profit. Recall that the central bank's flow profit is determined by the exchange rate ($g_t = F^* \Delta e_t$), and the exchange rate is related to inflation and output through Equation (7). It is straightforward to see that

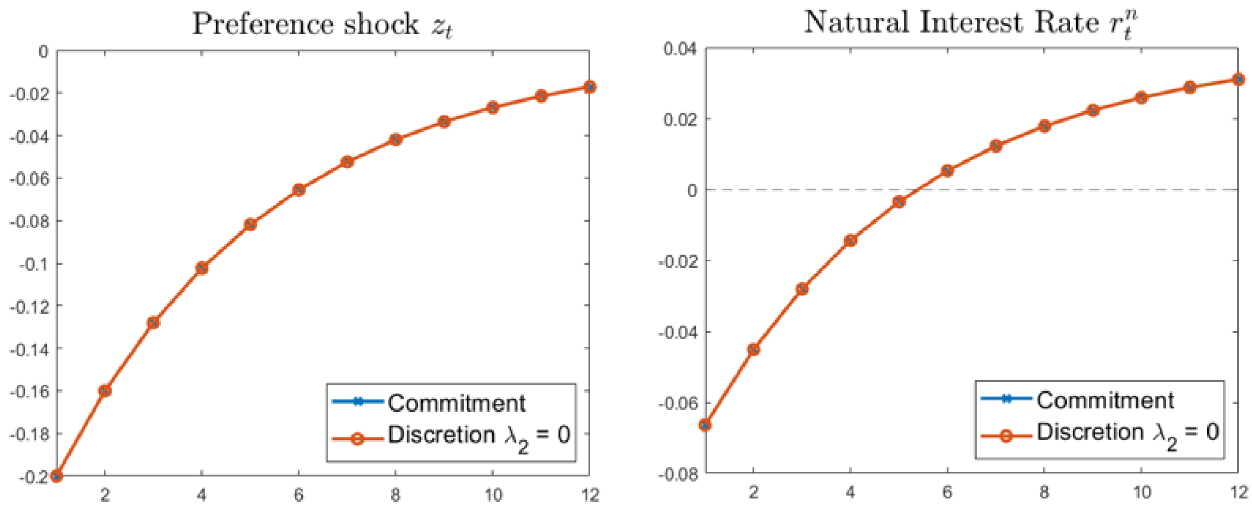
$$g_t = F^* (\pi_{H,t} + \tilde{y}_t) + t.i.p, \tag{13}$$

where $t.i.p$ represents terms independent of the discretionary central bank's policy. The above equation states that the central bank's flow profit is increasing in output gap and inflation. Therefore, the greater the central bank's profit concern, the more expansionary policy it would pursue, as captured by Equation (12).

5.2.2 Zero Lower Bound

The zero lower bound (ZLB) design is constructed as a negative shock on household preferences. At $t = 1$, there's an unexpected negative demand shock z_t, which follows an AR(1) process. The left panel of Figure 8 displays the demand shock z_t and its evolution. The shocks are identical to both types of central banks. After $t = 1$, all agents know the

Figure 8. Demand Shock and the Natural Interest Rate



The left panel displays the demand shock, and the right panel displays the natural interest rate. At $t = 1$, there's a negative demand shock, which follows an AR(1) process. The natural interest rate is a function of the demand shock and it becomes negative for five periods (from $t = 1, ..., 5$) and resumes to a positive level at $t = 6$.

subsequent path of z_t with certainty. Since the natural interest rate is a function of the demand shock and given the size of the shock, the natural interest rate becomes negative for five periods (from $t = 1, ..., 5$) and resumes to a positive level at $t = 6$. This is displayed in the right panel of Figure 8.

Note that in a standard New Keynesian model, it is always first-best to match the nominal interest rate to the natural interest rate. By doing so, the central bank can close the output and inflation gap simultaneously. However, when the natural interest rate is negative, the nominal interest rate cannot match it due to the ZLB constraint. As a result, the nominal interest rate is stuck at zero for five periods from $t = 1, ..., 5$. This is the ZLB scenario studied by this research.

5.2.3 Solving the Model

This paper utilizes the piece-wise linear solution provided by Guerrieri and Iacoviello (2015). In the current model, there are two sets of regime-switching conditions. The first is the ZLB constraint on the nominal interest rate, and the second is the asymmetric profit concerns. Denote the default regimes as the equilibrium when $i_t > 0$ and the equilibrium when ZLB is binding ($i_t = 0$) as the alternative regime. Similarly, define equilibrium with the non-negative accounting profit ($g_t \geq 0$) as the default regime and $g_t < 0$ as the alternative regime. After obtaining the equilibrium equations and log-linearizing the whole system, we can obtain the general solution of the form:

$$X_t = C(X_{t-1}, \epsilon_t; \Phi) + P(X_{t-1}, \epsilon_t; \Phi)X_{t-1} + Q(X_{t-1}, \epsilon_t; \Phi)\epsilon_t.$$

Note that policy function generally depends on whether the default regimes are prevailing. Guerrieri and Iacoviello (2015) propose an algorithm of looking forward to estimation and re-write the solution as:

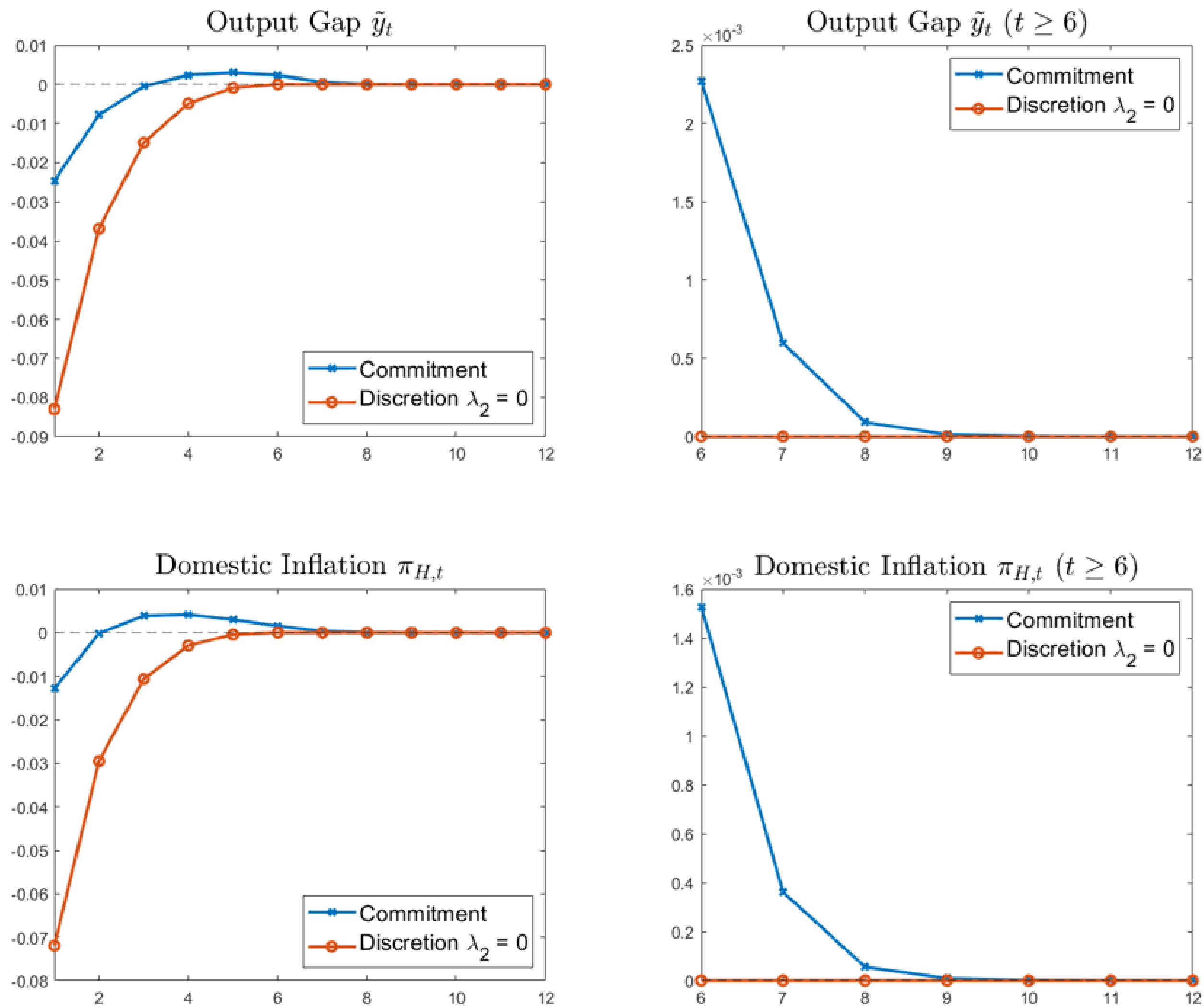
$$X_t = J(\boldsymbol{D}_t, \Phi) + H(\boldsymbol{D}_t, \Phi)X_{t-1} + G(\boldsymbol{D}_t, \Phi)\epsilon_t,$$

where X_t is a vector of endogenous variables and $\boldsymbol{D}_t = [ZLB_t, Profit_t]$ is the number of periods each alternative regime will prevail from date t. For instance, $\boldsymbol{D}_t = [5, 8]$ means that the ZLB will bind ($i_t = 0$) for five periods, and the accounting profit would be negative ($g_t < 0$) for eight periods from time t. Given a guess for $\boldsymbol{D}_t$, one can solve for time-varying coefficients in policy matrices. Then verify that the path of X_t is consistent with the guess. Using this method, I solved the model and obtained the piece-wise linear solution. The equilibrium allocations are displayed in the next section. For a more detailed description on the solution method and programming algorithm, see Appendix 7.4.

5.2.4 Optimal Escape

I start the analysis by looking at the two standard cases: the equilibrium allocations under discretionary and commitment central banks, *both without* profit concerns. The equilibrium allocations are provided in Figure 9. The line with circles represents the equilibrium allocation for the output gap and domestic inflation under the discretionary central bank. When the ZLB is binding ($t = 1, ..., 5$), there's a large negative output gap and deflation

Figure 9. Equilibrium Allocation for Commitment and Discretionary Central Banks with $\lambda_2 = 0$



This figure shows the equilibrium allocation for inflation and the output gap under the liquidity trap. The two left panels show that when the ZLB is binding ($t = 1, ..., 5$), there's a large negative output gap and deflation under the discretionary central bank with no profit concerns $\lambda_2 = 0$. The situation is much better with the central bank under commitment. This is because commitment central banks can commit to higher future inflation after $t > 5$ while discretionary central banks can't. This is demonstrated in the two right panels.

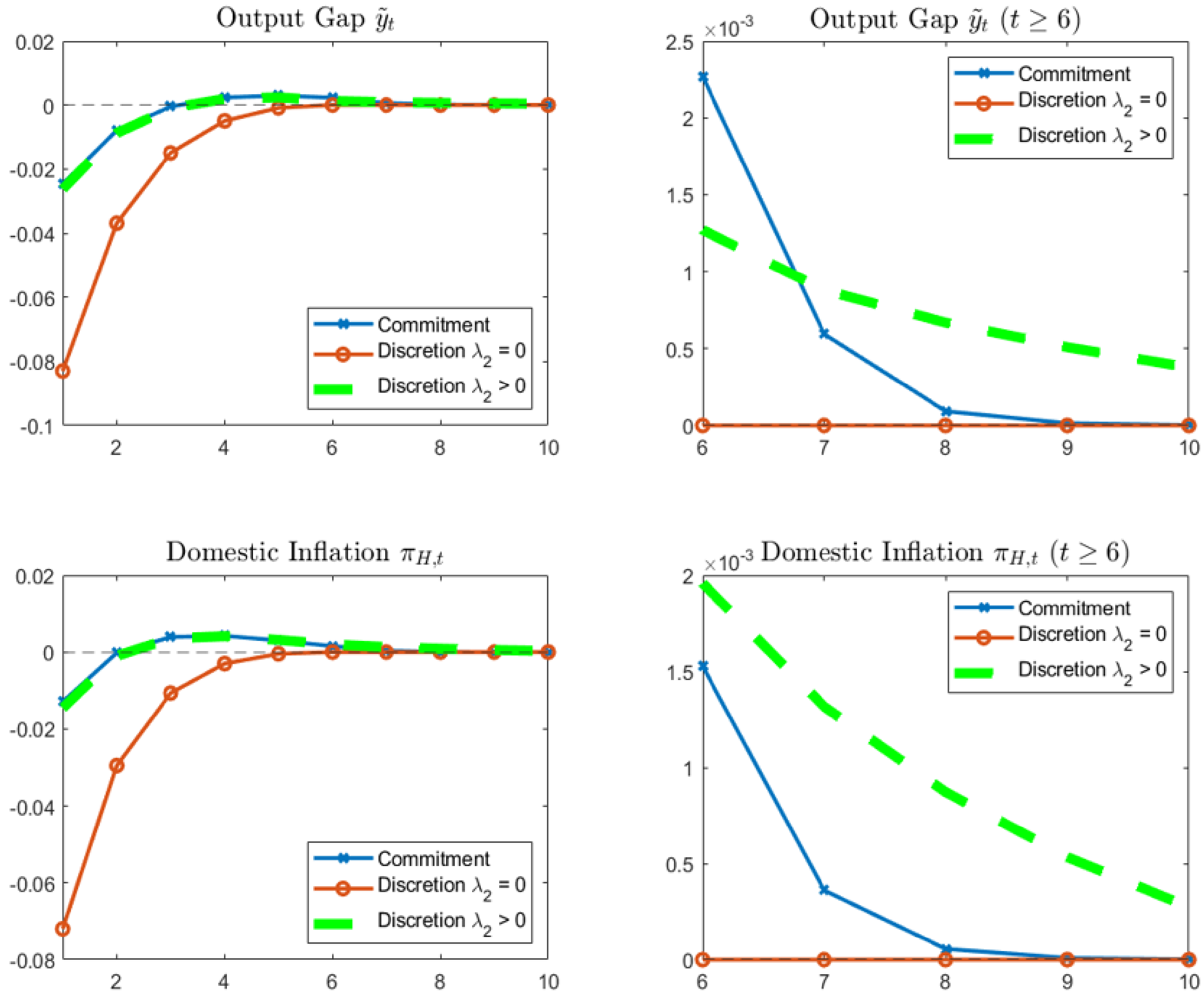
under discretionary central with no profit concerns ($\lambda_2 = 0$). The output and inflation gap is closed after $t \geq 6$ since the central bank can now set the interest rate equal to its natural level. The presence of the ZLB on the nominal interest rate is the ultimate source of welfare losses and the reason why the interest rate is higher than its optimal level. Those losses are considerably reduced when the central bank can commit to a policy plan. The line with crosses in Figure 9 is the equilibrium allocation under the commitment central bank. The inflation and output gap is much smaller compared to the discretionary central bank. What makes this possible is the central bank's commitment to higher inflation after the shock (after $t \geq 6$). This is displayed in the two right panel of Figure 9. Note that this commitment is not time-consistent — If the commitment central bank can re-optimize at $t = 6$, it will renege on its earlier promises and close the output and inflation gap as the discretionary central bank does. The findings here are consistent with Galí (2015).

This section aims to show that central bank profit concerns can be used as a commitment device to help escape a liquidity trap. I achieve this goal by showing that when the ZLB is binding ($t = 1, ..., 5$), the equilibrium allocation for the output gap and domestic inflation under the discretionary central bank *with profit concerns* are numerically similar to the commitment central bank. The results are displayed in Figure 10. The dotted line is the equilibrium path of a discretionary central bank *with* profit concerns ($\lambda_2 > 0$) and resembles the commitment central bank. This is because discretionary central banks with profit concerns would have an inflation bias and, therefore, can make a time-consistent commitment to raise inflation after the shock (after $t \geq 6$). This credible commitment helps the economy escape a liquidity trap.

The key to the discretionary central bank's time-consistent commitment to raise inflation is its impact on profit, which is captured by Equation (13). When the negative demand shock hits, the economy experiences deflation and local currency appreciation. As a result, the central bank's accounting profit is negative due to the local currency's appreciation. The central bank thus has an incentive to cause inflation (and therefore local currency depreciation) to minimize the losses, since a higher inflation and output gap increases profit by Equation (13).

Also note that by Equation (12), the discretionary central bank with profit concerns would have an inflation bias whenever $g_t < 0$. This means it would pursue a more inflationary policy compared to the bank without profit concerns. This is captured by the two right panels of Figure 10. As the losses decrease and approach zero over time, the inflation bias

Figure 10. Equilibrium Allocation for Commitment and Discretionary Central Banks with $\lambda_2 > 0$



This figure shows the equilibrium allocation for inflation and the output gap under the liquidity trap. The two left panels show that when the ZLB is binding $(t = 1, ..., 5)$, there's a large negative output gap and deflation under discretionary central with no profit concerns $\lambda_2 = 0$. The situation is much better under the commitment central bank and the discretionary central bank with profit concerns $\lambda_2 > 0$. This is because discretionary central banks with profit concerns would have an inflation bias and, therefore, can make a time-consistent commitment to raise inflation after the shock. Consequently, its equilibrium allocation for inflation and output gap when the ZLB is binding $(t = 1, ..., 5)$ resembles the commitment central bank.

also decreases. To summarize, if the discretionary central bank has profit concerns $(\lambda_2 > 0)$, it has the incentive to keep inflation high after the crisis is over (after $t \geq 6$) in a time-consistent manner. Therefore, its equilibrium allocation for inflation and output gap when the ZLB is binding $(t = 1, ..., 5)$ resembles the commitment central bank.

The result shown here depends on the key relationship that expansionary monetary policy leads to an accounting profit for the central bank. Therefore, a more complicated assumption on the central bank's balance sheet and on the model setting should not change the main results as long as the key relationship mentioned above still stands.

5.3 Simulation and Welfare Analysis

In this section, data are simulated from the model to perform welfare analysis. I follow the approach provided by Rotemberg and Woodford (1999), who constructs a welfare-based criterion relying on a second-order approximation to the utility losses of the representative household. The approximation of the representative household's utility yields the following welfare loss function:

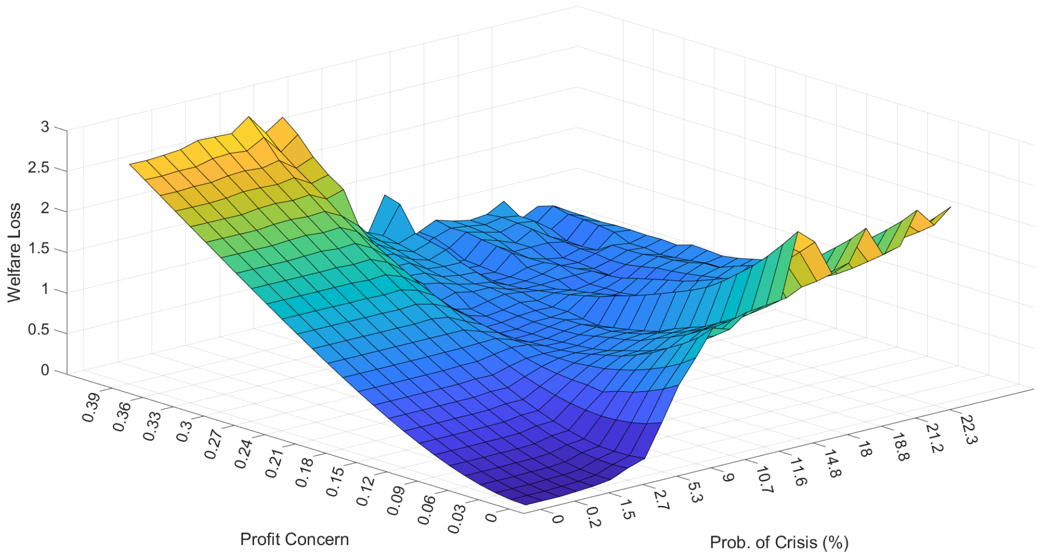
$$\begin{aligned}\mathbb{W} &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U(C_t, N_t; Z_t) - U(C, N; Z)}{U_c(C, N; Z)C} \right) \\ &= \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda}\pi_t^2 \right],\end{aligned}$$

where losses are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady-state consumption. The average welfare loss per period is thus given by a linear combination of the variance of the output gap and inflation:

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) var(\tilde{y}_t) + \frac{\epsilon}{\lambda} var(\pi_t) \right]. \tag{14}$$

Data are simulated for different profit concerns (λ_2) and crisis probabilities. The crisis probabilities are defined by the probability that the natural interest rates are negative (and the ZLB on the nominal interest rate is binding). Recall that the demand shock follows an AR(1) process, $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ with $\varepsilon_t^z \sim N(0, \sigma_z)$. By increasing σ_z, the natural interest rates become more likely to take on large negative values, leading to longer binding periods of ZLB constraint. Figure 11 shows the welfare loss defined in Equation (14) for different combinations of profit concerns (λ_2) and crisis probabilities. The plot indicates that when

Figure 11. Welfare Loss from the Simulation

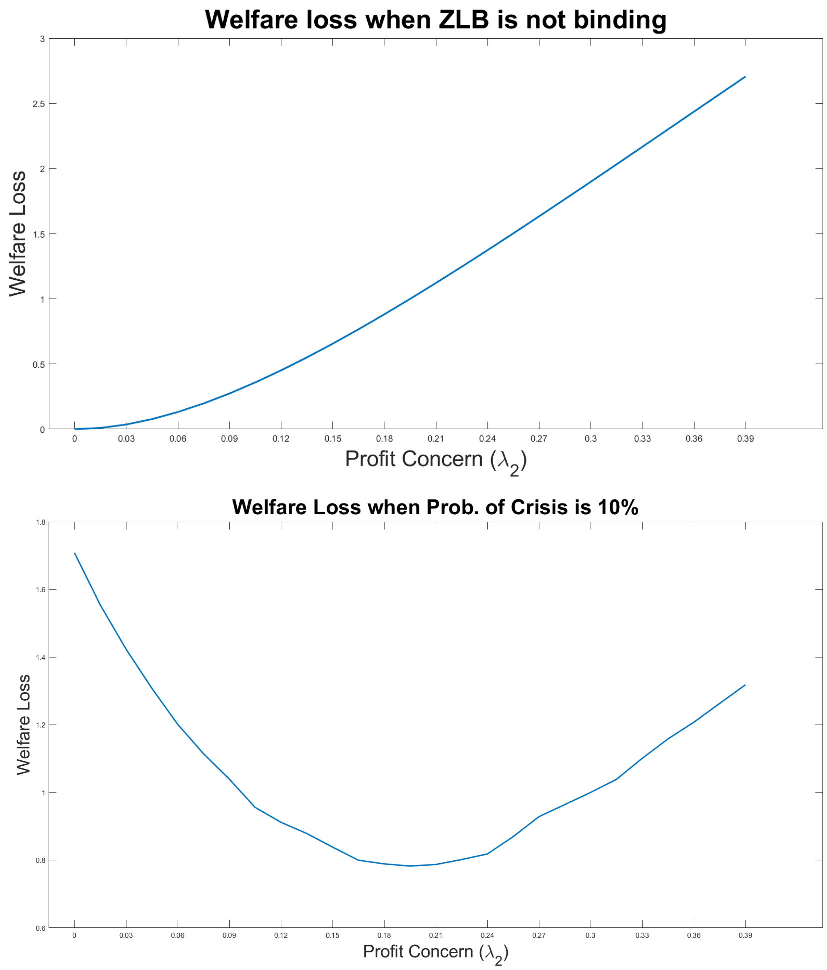


This figure shows the welfare loss defined in Equation (14) for different combinations of profit concerns (λ_2) and crisis probabilities. The crisis probabilities are defined by the probability that the ZLB on the nominal interest rate is binding.

the ZLB constraint is not binding (crisis probability $= 0$), the welfare loss monotonically increases in profit concerns (λ_2). However, when the crisis probability increases, $\lambda_2 = 0$ no longer yields the optimal results. For a crisis probability large enough, welfare loss becomes a U-shape function of λ_2. Welfare loss decreases in λ_2 to a certain point and then increases in λ_2. Figure (12) demonstrates this point and shows that when the economy has a positive change to be stuck in the liquidity trap, $\lambda_2 = 0$ no longer yields the optimal outcome.

The intuition is straightforward. First, consider the case when the ZLB on the nominal interest rate is never binding. Under such a scenario, the central bank can always set the interest rate to its optimal level and simultaneously close the output and inflation gap. Therefore, profit concerns are simply a "distraction" for the central bank and cause the inflation and output gap to be higher than optimal. As a result, the welfare loss monotonically increases with profit concern (λ_2). On the other hand, when the ZLB on the nominal interest rate is binding from time to time, profit concerns *can* be welfare increasing. When the ZLB is not binding, profit concerns increase inflation and output gap as before, which decreases

Figure 12. Sectional View of the Welfare Loss from the Simulation



This figure shows the sectional view of welfare loss defined in Equation (14) for different profit concerns (λ_2) and crisis probabilities. Crisis probabilities are defined by the probability that the ZLB on the nominal interest rate is binding. The top panel displays the welfare loss for various profit concerns (λ_2) given that the ZLB is never binding; The bottom panel displays the welfare loss for various profit concerns (λ_2) given that the ZLB on the nominal interest rate is binding 10% of the time.

welfare. However, when the ZLB is binding and the economy is in a liquidity trap, profit concerns decrease deflation and the negative output gap, which increases welfare. As a result, how profit concerns affect welfare depends on how frequently the ZLB binds and how large the profit concerns are. As the probability of crisis increases, the benefit of profit concerns also increases. To sum up, under normal circumstances (ZLB is not binding), central bank profit concerns are welfare decreasing. However, when the probability of a crisis is greater than zero, Figure (11) and (12) together show that the optimal profit concern λ_2 is greater than zero. This indicates that profit concerns can increase welfare.

6 Conclusion

This paper explores the relationship between monetary policy and central bank profit concerns. Using a panel of 116 central banks from 2000 to 2024, I first demonstrate how central banks' profits increase when their local currency depreciates. Second, I show that central banks intervene in the foreign exchange market right before releasing financial statements. These interventions are due to profit concerns and are welfare-reducing in ordinary circumstances. However, using a simple New Keynesian model, I demonstrate that when the nominal interest rate is at the zero lower bound, central bank profit concerns can be welfare-increasing — profit concerns can serve as a commitment device and provide an optimal escape from the liquidity trap.

Two points are worth mentioning regarding how profit concerns can be welfare-increasing. First, the model implicitly assumes that the central bank's profit concerns are public knowledge. If the private sector is unaware of the central bank's profit motive, the mechanism shown in this paper wouldn't work. Therefore, the central bank needs to declare its profit concerns to the public for them to be welfare-increasing during a liquidity trap. Second, profit concerns help the economy by providing an optimal escape from the liquidity trap. For most developing countries where low nominal interest rates are never a problem, profit concerns are always welfare-decreasing. For developed countries, on the other hand, with many of them in the liquidity trap during the COVID crisis, profit concerns are another unconventional monetary policy that the central bank can consider.

This paper could be the basis for several future studies. First, central banks have multiple ways to increase their reported profit, including manipulating domestic interest rates. This research only identifies *one* way the central banks use to increase profit — through intervening in the foreign exchange market. Other policy tools and actions taken by central banks due to profit motives could also be important and worth investigating. Second, this paper only identifies that central banks intervene in the foreign exchange market for profit motives. How effective those interventions are is not something this paper can answer. This paper hopes to inspire future works on central bank profit concerns and help understand how much central bank care about profit and what actions they are willing to take to avoid reporting losses.

7 Appendix

7.1 The Distortion Pattern with Conventional FXI Measurements

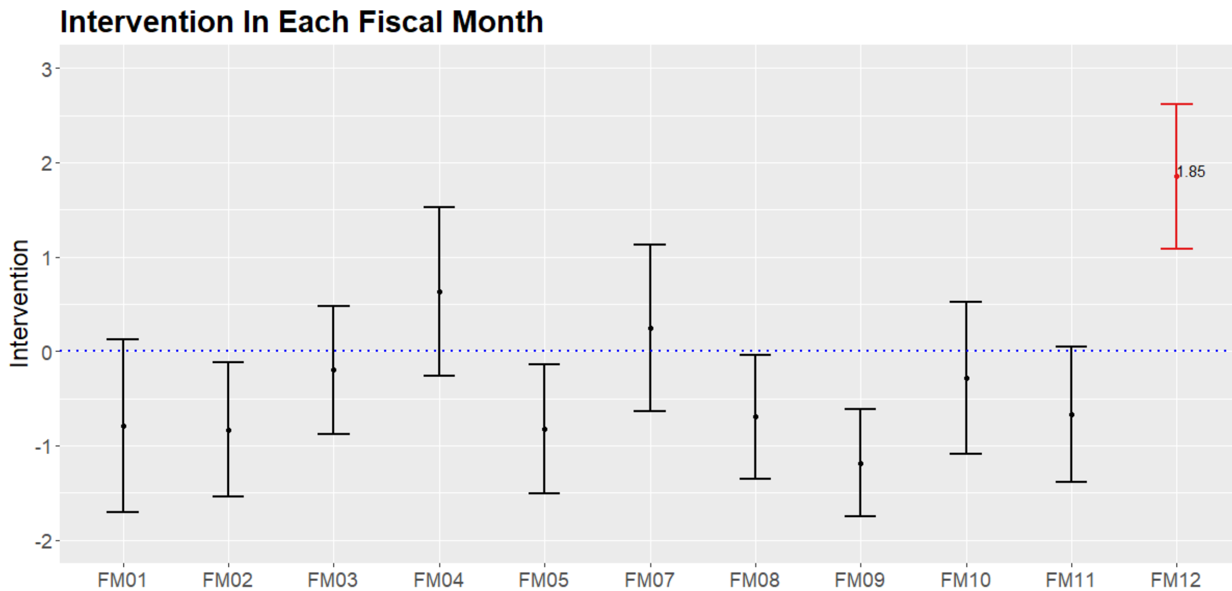
The intervention measurement used in this research is the central banks' monthly percentage change in foreign reserves (measured in USD). Formally, the intervention is measured by

$$Y_{i,t,m} = \frac{Reserve^{USD}_{i,t,m} - Reserve^{USD}_{i,t,m-1}}{|Reserve^{USD}_{i,t,m-1}|},$$

where $Reserve^{USD}_{i,t,m}$ is the value in USD of the net foreign assets for central bank i at the end of year t and month m. By increasing $Y_{i,t,m}$, central bank i can create depreciation pressure on local currency on year t month m. Moreover, $Y_{i,t,m}$ is trimmed at the 1^{st} and 99^{th} percentiles to control for outliers.

We can see how central banks' intervention $Y_{i,t,m}$ differ across each *fiscal* month by estimating equation (|1). Figure 13 displays the estimations. The intervals represent a 95% confidence ban, and the standard errors are clustered for central banks. The point estimation for β_{12} is 1.85 and is significant at the 1% level. This means that the intervention $Y_{i,t,m}$ is 1.85 higher in the last fiscal month compared to the middle of the fiscal year (the unconditional standard deviation for $Y_{i,t,m}$ is 6.80. Therefore, 1.85 is about 0.3 standard deviations). Central banks intervened aggressively in the last fiscal month, captured by the positive and significant β_{12}. This positive intervention would cause depreciation pressure on the local currency and help the central bank to report a higher profit.

Figure 13. Foreign Exchange Market Intervention In Each Fiscal Month Compared to the Middle of the Fiscal Year



This figure shows the estimation results for equation (1). 95% confidence intervals are displayed and standard errors are clustered for central banks. The key parameter of interest β_{12} is significant with the point estimate of 1.85. *Data Sources:* IMF's International Financial Statistics.

7.2 Model

We start with problems facing households and firms. Before we do so, a brief remark on notation is needed. A typical variable at time t is of the upper case with subscript t (e.g., X_t). Unless specified otherwise, a variable without the subscript t would be denoted as the steady state value of that variable (e.g., X). A lower case of a variable represents the log deviation from the steady state (e.g., $x_t \equiv \ln(X_t/X)$). All log-linearizations are around a zero inflation steady state.

7.2.1 Households

By the structure of the CES function, we can show that the optimality behavior requires that for all domestic goods $i \in [0, 1]$, the demand schedule is given by:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t},$$

and combing the above expression with the definitions of $P_{H,t}$ and $C_{H,t}$ yield

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di = P_{H,t}C_{H,t}.$$

Similarly, it can be shown that the optimal allocation between $C_{H,t}$ and $C_{F,t}$ is given by:

$$P_{H,t}C_{H,t} = (1-\nu)P_tC_t, \quad P_{F,t}C_{F,t} = \nu P_tC_t. \tag{15}$$

It is then straightforward to show that the period budget constraint for households can be simplified to:

$$P_tC_t + \mathbb{E}_t\left[Q_{t,t+1}D_{t+1}\right] \leq D_t + W_tN_t - T_t + \Lambda_t. \tag{16}$$

Under the assumption of perfectly competitive labor markets, the household's intratemporal optimality condition is given by:

$$C_tN_t^{\varphi} = \Omega_t, \tag{17}$$

where $\Omega_t \equiv W_t/P_t$ represents the real wage. This represents the household's labor supply condition and the log-linearized form is

$$\boldsymbol{\omega}_t = c_t + \varphi n_t \tag{18}$$

For the household's intertemporal optimality condition, the following equation must hold for all possible state ξ in time $t+1$ due to the complete market assumption:

$$\begin{aligned} &\frac{V_{t,t+1}(\xi)}{P_t}\frac{Z_t}{C_t} = \beta\frac{\eta(\xi',\xi)}{P_{t+1}}\frac{Z_{t+1}}{C_{t+1}} \\ \Rightarrow\ &\frac{V_{t,t+1}(\xi)}{\eta(\xi',\xi)} = \beta\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{Z_{t+1}}{Z_t}\right), \end{aligned} \tag{19}$$

where $V_{t,t+1}(\xi)$ is the period t price (in domestic currency) of an Arrow security that pays one unit of domestic currency in state ξ and nothing otherwise. $\eta(\xi', \xi)$ is the probability of transitioning from the current state ξ' to the next period state ξ, $C_{t+1} \equiv C_{t+1}(\xi)$ is the consumption level in $t+1$ at state ξ. Similar definition applies for P_{t+1} and Z_{t+1}. The above equations mean that the utility loss from the purchase of an Arrow security must equal the expected one-period-ahead utility gain from the additional consumption made possible by the eventual security payoff. Moreover, the price of an Arrow security must satisfy $V_{t,t+1}(\xi) = \eta(\xi', \xi) Q_{t,t+1}$. Therefore, we have:

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right) \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right),$$

which is assumed to be satisfied for all possible states of nature at t and $t+1$. Taking conditional expectation would yield the conventional stochastic Euler equation:

$$Q_t = \beta \mathbb{E}_t \left[\left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right], \tag{20}$$

where $Q_t = \mathbb{E}_t[Q_{t,t+1}]$ denotes the price of a one-period discount bond paying off one unit of domestic currency in all states at $t+1$. The log-linearized form is given by:

$$c_t = \rho + \mathbb{E}_t[c_{t+1}] + \mathbb{E}_t[\pi_{t+1}] + (1 - \rho_z) z_t - i_t, \tag{21}$$

where $i_t \equiv -\log Q_t$ is the short term nominal rate, $\rho \equiv -\log \beta$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is the CPI inflation.

7.2.2 Firms

All firms $i \in [0, 1]$ are assumed to face an identical demand schedule given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_t, \tag{22}$$

where domestic price index $P_{H,t}$ and aggregate output Y_t are taken as given by all firms. Given the demand schedule, the price will determine the output sold, and the labor hired will be determined through the production function. Therefore, there's only one decision that firms need to make, which is deciding $P_{H,t}(i)$.

Let $S(t)$ represents the set of firms that can not reoptimize. By the fact that all firms that can reoptimize will choose an identical price $\bar{P}_{H,t}$, we have:

$$\begin{aligned} P_{H,t} &= \left[\int_{S(t)} (P_{H,t-1}(i))^{1-\epsilon} di + (1-\theta)(\bar{P}_{H,t})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\ &= \left[\theta(P_{H,t-1})^{1-\epsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \end{aligned}$$

where the second equality comes from the independent assumption. The log-linear approximation around a zero inflation steady state is given by

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta)\bar{p}_{H,t}. \tag{23}$$

A firm reoptimizing in period t will choose $P_{H,t}(i) = \bar{P}_{H,t}$ that maximizes the current market value of the profits generated while that price remains effective. This corresponds to solving:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}}\right) \left(\bar{P}_{H,t} Y_{t+k|t} - TC_{t+k|t}\right)\right],$$

subject to the period demand constraints:

$$Y_{t+k|t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{24}$$

where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t} = \beta^k (Z_{t+k}/Z_t)(C_t/C_{t+k})$ is the stochastic discount factor, $Y_{t+k|t}$ is the output in period $t+k$ for a firm that last set its price at period t, and $TC_{t+k|t}$ is the nominal total cost at time $t+k$ for a firm that last set its price at period t. It is always assumed that the firm meets the demand for its goods at the current price.

FOC with respect to $\bar{P}_{H,t}$ yield:

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(\frac{Y_{t+k|t}}{P_{t+k}} \right) \left(\bar{P}_{H,t} - \frac{\epsilon}{\epsilon - 1} MC_{t+k|t} \right) \right] = 0, \tag{25}$$

where $MC_{t+k|t}$ is the nominal marginal cost in period $t+k$ for a firm that last set its price at period t and is given by $MC_{t+k|t} = (1+\tau)W_{t+k}$. Note that the marginal cost is common across all firms regardless of previous price settings due to the property of constant return to scale of the production technology. Log-linearize the above expression would yield:

$$\bar{p}_{H,t} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t[\boldsymbol{\omega}_{t+k} + p_{t+k}]. \tag{26}$$

The forward looking price dynamics (26) can be rewritten as the following recursive form:

$$\bar{p}_{H,t} = \beta\theta \mathbb{E}_t[\bar{p}_{H,t+1}] + (1 - \beta\theta)(\boldsymbol{\omega}_t + p_t)$$

provided that $\lim_{k\to\infty} (\theta\beta)^k \mathbb{E}_t[\bar{p}_{H,t+k}] < \infty$, which is guaranteed since the steady state inflation is zero. Combine the above with the definition of CPI and aggregate price dynamics (23) yield a version of the New Keynesian Phillips curve:

$$\pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \lambda(\boldsymbol{\omega}_t + \nu s_t), \tag{27}$$

where $\lambda \equiv (1 - \beta\theta)(1 - \theta)/\theta$.

7.2.3 Identities in the Small Open Economy

Combine the definition of terms of trade and the law of one price and log-linearize would yield:

$$e_t = s_t + p_{H,t}. \tag{28}$$

Recall that for domestic households, the following equation must hold for all state $\xi \in \Xi$

in time $t+1$ due to the complete market assumption:

$$\frac{V_{t,t+1}(\xi)}{\eta(\xi',\xi)} = \beta\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{Z_{t+1}}{Z_t}\right).$$

Under the assumption of a complete set of state-contingent securities traded internationally, a condition analogous to the above equation must hold for foreign households. Assume that foreign households have the same utility function as domestic ones, except that demand shock is not present for foreign households. We then have:

$$\begin{aligned} \frac{V_{t,t+1}(\xi)}{\mathcal{E}_t P_t^*}\frac{1}{C_t^*} &= \beta\frac{\eta(\xi',\xi)}{\mathcal{E}_{t+1}P_{t+1}^*}\frac{1}{C_{t+1}^*} \\ \Rightarrow \frac{V_{t,t+1}(\xi)}{\eta(\xi',\xi)} &= \beta\left(\frac{C_t^*}{C_{t+1}^*}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right), \end{aligned} \tag{29}$$

The above equations mean that the utility loss from purchasing an Arrow security for international households must equal the expected one-period-ahead utility gain from the additional consumption made possible by the eventual security payoff.

Combine equation (19) and (29), together with the definition of terms of trade and the law of one price yield the international risk sharing condition:

$$C_t = \vartheta C_t^* \mathcal{S}_t^{1-\nu} Z_t \tag{30}$$

for all t, and where ϑ is a constant, which will depend on the initial conditions regarding relative net asset positions. Henceforth, symmetric initial conditions are assumed, implying $\vartheta = 1$. Thus, the model assumption leads to a simple relationship linking domestic and world consumption. The risk-sharing condition can be log-linearized as

$$c_t = c_t^* + (1-\nu)\, s_t + z_t. \tag{31}$$

Finally, denote Q_t^* as the price of a risk-free bond that yields one unit of foreign currency in all states. Then, under the assumption that the foreign households have the identical utility function as the domestic ones, a foreign version of equation (20) is given by

$$Q_t^* = \beta \mathbb{E}_t \left[\frac{C_t^*}{C_{t+1}^*} \right].$$

Combine this with equation (20), as well as the international risk-sharing condition would yield the familiar uncovered interest parity (UIP) property:

$$i_t = i_t^* + \mathbb{E}_t[\Delta e_{t+1}], \tag{32}$$

where $i_t^* \equiv -\ln(Q_t^*)$ is the international interest rate.

7.2.4 Market Clearing

The domestic goods market clearing condition in the home economy requires:

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + X_t(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} \left[(1-\nu) \left(\frac{P_t}{P_{H,t}} \right) C_t + \nu \mathcal{S}^\nu Y_t^* \right] \quad \forall\, i \in [0,1]. \end{aligned}$$

Integrate the above expression over goods $i \in [0,1]$ yields

$$Y_t = (1-\nu) \left(\frac{P_t}{P_{H,t}} \right) C_t + \nu \mathcal{S}_t Y_t^*.$$

The log-linearized (around the symmetric steady state) version is given by:

$$y_t = (1-\nu)c_t + \nu(2-\nu)s_t + \nu c_t^*. \tag{33}$$

For the domestic labor market to clear, we need the aggregate labor supply by households

N_t equal the sum of labor demand across firms:

$$N_t = \int_0^1 N_t(i)di = \int_0^1 Y_t(i)di$$
$$= Y_t \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di.$$

The second equation utilizes the demand schedule (22). Note that $\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di$ is a measure of price dispersion across firms. It can be shown that $\log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di = 0$ up to a first-order approximation around a symmetric steady state ($P_H(i) = P \ \ \forall i$). Thus, up to a first-order approximation, we have:

$$y_t = n_t. \tag{34}$$

7.2.5 Natural Level of Outputs

Note that under flexible pricing ($\theta = 0$), the optimality condition for intermediate firms become

$$\frac{\bar{P}_{H,t}}{P_t} = \frac{\epsilon(1-\tau)}{\epsilon - 1}\Omega_t, \tag{35}$$

where Ω_t is the firms' common marginal cost and $\epsilon(1-\tau)/(\epsilon-1)$ is the fixed markup common across all firms. Rearranging and using the definition of CPI yield:

$$\frac{\epsilon(1-\tau)}{\epsilon - 1} = \mathcal{S}_t^{-\nu}\Omega_t^{-1}$$

Note that this means that the relationship between $\mathcal{S}_t$ and Ω_t is determined by the equation above under flexible price ($\theta = 0$). Log-linearize would yield:

$$0 = \nu s_t^n + \boldsymbol{\omega}_t^n, \tag{36}$$

where the variables with superscript n represent the natural level. Moreover, from equation (18), (31), (33), and (34) we have:

$$\boldsymbol{\omega}_t^n = c_t^n + \varphi n_t^n \tag{37}$$

$$c_t^n = (1-\nu)s_t^n + z_t \tag{38}$$

$$y_t^n = (1-\nu)c_t^n + \nu(2-\nu)s_t^n \tag{39}$$

$$y_t^n = n_t^n. \tag{40}$$

We now have five unknowns $(n_t^n, \boldsymbol{\omega}_t^n, s_t^n, c_t^n, y_t^n)$ and five linearly independent equations (equations 36 to 40). We can now solve the natural levels in terms of exogenous shocks and obtain:

$$y_t^n = -\frac{\nu}{1+\varphi} z_t.$$

Finally, for the natural interest rate r_t^n, note that as shown in Galí (2015),

$$r_t^n = \mathbb{E}_t[\Delta y_{t+1}^n] + (1-\nu)(1-\rho_z)z_t.$$

Plug the solution of y_t^n into the above expression would yield:

$$r_t^n = (1-\rho_z)\phi z_t.$$

7.2.6 Equilibrium Conditions

This section derives the key equilibrium conditions (5), (6), and (7) used in the main text. I start with the Euler equation (5), derived from the household's optimality conditions. Note that from the household Euler equation (21) we have (ignoring the constant term):

$$c_t = \mathbb{E}_t[c_{t+1}] + \mathbb{E}_t[\pi_{t+1}] - i_t + (1-\rho_z)z_t.$$

Combing the above equation with $\pi_t = \pi_{H,t} + \nu\Delta s_t$ (the definition of CPI inflation), the goods market clearing condition (33), the international risk-sharing condition (31), and the fact that $r_t^n = (1-\rho_z)\phi z_t$ yields the dynamic IS curve for the small open economy:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - (i_t - \mathbb{E}_t[\pi_{H,t+1}] - r_t^n).$$

The New Keynesian Phillips curve (6) is derived by combining an earlier version of the Phillips curve (27) with Equation (18), (31), (33), and (34). After some algebra yield

$$\pi_{H,t} = \beta\mathbb{E}_t[\pi_{H,t+1}] + \lambda(1+\varphi)\tilde{y}_t.$$

Finally, the relationship between depreciation, inflation, and output, namely Equation (7), is obtained by combining Equation (28), (31), (33). This yield:

$$\Delta e_t = \tilde{y}_t - \tilde{y}_{t-1} + \pi_{H,t} - \phi(z_t - z_{t-1}).$$

7.3 Proof of Propositions

7.3.1 Proposition 1

The central bank under commitment is assumed to be able to commit, with full credibility, to a *policy plan*. In the model context, such a policy plan consists of a specification of the desired levels of inflation and output at all possible dates and states of nature.

the central bank chooses the policy plan for output gap and inflation, $\{\tilde{y}_t, \pi_{H,t}\}_{t=0}^{\infty}$ by solving the following problem:

$$\text{Min} \sum_{t=0}^{\infty} \beta^t(\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2)$$

$$s.t.\ \ \pi_{H,t} = \beta\mathbb{E}[\pi_{H,t+1}] + \kappa_1\tilde{y}_t$$

The associate Lagrangian is given by:

$$\begin{aligned}\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2 + \lambda_t \left(\pi_{H,t} - \beta \mathbb{E}_t[\pi_{H,t+1}] - \kappa_1 \tilde{y}_t\right)\right] \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2 + \lambda_t \left(\pi_{H,t} - \beta \pi_{H,t+1} - \kappa_1 \tilde{y}_t\right)\right] \quad \text{(by LIE)}.\end{aligned}$$

Differentiate the Lagrangian with respect to y_t and $\pi_{H,t}$, and eliminate the Lagrange multiplier would yield the optimal condition

$$\tilde{y}_t - \tilde{y}_{t-1} = -\frac{\kappa_1}{\lambda_1} \pi_{H,t}, \tag{41}$$

for $t = 0, 1, 2, ...$ and $\tilde{y}_{-1}$ is predetermined and assumed to be zero. Note that we can rewrite the above equation in level terms:

$$\tilde{y}_t = -\frac{\kappa_1}{\lambda_1}(p_{H,t} - p_{H,-1}) \equiv -\frac{\kappa_1}{\lambda_1} \hat{p}_{H,t}, \tag{42}$$

where $\hat{p}_{H,t} \equiv (p_{H,t} - p_{H,-1})$ is the (log) deviation between the price level and an "implicit target" given by the price level prevailing one period before the central bank. Without loss of generality, normalize the price level prevailing in $t = -1$ to be one. We then have $\hat{p}_{H,t} = p_{H,t}$. Reorganize the above equation would yield proposition 1:

$$p_{H,t} = -\frac{\lambda_1}{\kappa_1} \tilde{y}_t.$$

7.3.2 Proposition 2

The discretionary central bank cannot commit to any future actions and is assumed to make whatever decision is optimal at the time. Therefore, the bank faces a sequential optimization problem. Each period the central bank chooses $\{y_t, \pi_{H,t}\}$ to minimize

$$\begin{aligned}&\text{Min } \pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2 + \lambda_2 g_t^2 \cdot \mathbb{I}(g_t < 0)\\ &s.t.\ \ (6) \sim (8).\end{aligned}$$

The solutions are derived under two different scenarios. First, consider the case when the accounting flow profit is non-negative $g_t \geq 0$. In this case, the third term in the central bank's objective function is dropped, and the bank solves the following problem:

$$\begin{aligned}&\text{Min } \pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2\\ &s.t.\ \pi_t = \kappa_1 \tilde{y}_t + v_t,\end{aligned}$$

where $v_t \equiv \beta \mathbb{E}_t[\pi_{H,t+1}]$ is taken to be given by the monetary authority. Note that $\mathbb{E}_t[\pi_{H,t+1}]$ is a function of expectation on the future output gap and cannot be influenced by the central bank due to the assumption of no commitment power. The optimal condition can be derived easily and is given by:

$$\pi_{H,t} = -\frac{\lambda_1}{\kappa_1} \tilde{y}_t.$$

On the other hand, when the accounting flow profit is negative $g_t < 0$, the central bank solves the following problem:

$$\begin{aligned}&\text{Min } \pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2 + \lambda_2 g_t^2\\ &s.t.\ \ (6) \sim (8).\end{aligned}$$

It is straightforward to see that the optimality condition yields:

$$\pi_{H,t} = -\frac{\lambda_1}{\kappa_1} \tilde{y}_t - \lambda_2 \frac{\kappa_2}{\kappa_1} g_t.$$

7.4 Solution Methods: Solving the Model with Commitment Central Bank

This model deals with two sets of regime-switching conditions and utilizes the piecewise linear solution provided Guerrieri and Iacoviello (2015). The complete solution method should be referred to the original Guerrieri and Iacoviello (2015) paper. However, here I present a detailed procedure for solving the model with the central bank under commitment and with no profit concerns. Since, without the profit concern, there is only one set of regime-switching conditions in this case, it's easier to characterize. The solution methods provided here are based on Galí (2015).

In period $t = 0$, when the unexpected drop in r_t^n is realized, the central bank chooses the policy plan for output gap and inflation, $\{\tilde{y}_t, \pi_{H,t}\}_{t=0}^{\infty}$ by solving the following problem

$$\text{Min} \sum_{t=0}^{\infty} \beta^t (\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2)$$

$$s.t.\ \ \pi_{H,t} = \beta \pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t \quad \text{and} \quad i_t \geq 0.$$

First, note that since the path for z_t is assumed to be known by all agents after the unexpected shock in $t = 0$. Therefore, the model has no uncertainty, and the expectation operator drops out. Moreover, since the central bank has full credibility, the central bank can determine future variables such as $\pi_{H,t+1}$. Specifically, it can commit to a policy plan and follow through. The Lagrangian for the above problem takes the form:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}(\pi_{H,t}^2 + \lambda_1 \tilde{y}_t^2) + \xi_{1,t}(\pi_{H,t} - \lambda(1+\varphi)\tilde{y}_t - \beta\pi_{H,t+1}) + \xi_{2,t}(\tilde{y}_t - \tilde{y}_{t+1} - \pi_{H,t+1} - r_t^n) \right]$$

with the corresponding first order conditions, slackness conditions, and initial conditions:

$$\pi_{H,t} + \xi_{1,t} - \xi_{1,t-1} - \frac{1}{\beta}\xi_{2,t-1} = 0 \tag{43}$$

$$\lambda_1 \tilde{y}_t - \lambda(1+\varphi)\xi_{1,t} + \xi_{2,t} - \frac{1}{\beta}\xi_{2,t-1} = 0 \tag{44}$$

$$\xi_{2,t} \geq 0; \quad i_t \geq 0 \quad \text{(with complementary slackness)} \tag{45}$$

$$\xi_{1,-1} = \xi_{2,-1} = 0. \tag{46}$$

The equilibrium path for i_t is conjectured and later verified to be in the following form.

$$\begin{cases} i_t = 0 \;\; \text{(which imply } \xi_{2,t} > 0) & \text{for } t = 0, ..., t_C \\ i_t > 0 \;\; \text{(which imply } \xi_{2,t} = 0) & \text{for } t = t_C + 1, ..., \end{cases}$$

where $t_C \geq t_Z$ is the last period that $i_t = 0$. Note that the periods for ZLB to bind t_C could potentially be longer than the periods of negative shocks t_Z.

We first focus on the equilibrium allocation for period t_C+2 onward. For $t = t_C+1, t_C+2, ...$, given the conjecture for the equilibrium path for i_t, we know that $\xi_{2,t} = 0$. Plug this into the FOC and combine with the New Keynesian Phillips curve (NKPC) we have

$$\pi_{H,t} + \xi_{1,t} - \xi_{1,t-1} = 0$$

$$\lambda_1 \tilde{y}_t - \lambda(1+\varphi)\xi_{1,t} = 0$$

$$\pi_{H,t} = \beta\pi_{H,t+1} + \lambda(1+\varphi)\tilde{y}_t$$

Note that this is a system of three differential equations with three unknown, and can be written as

$$\begin{bmatrix} \pi_{H,t} \\ \xi_{1,t} \\ \tilde{y}_t \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} \pi_{H,t-1} \\ \xi_{1,t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \boldsymbol{D} \begin{bmatrix} \pi_{H,t+1} \\ \xi_{1,t+1} \\ \tilde{y}_{t+1} \end{bmatrix}$$

Using method of undetermined coefficient, we can obtain the solution of the form

$$\begin{bmatrix} \pi_{H,t} \\ \xi_{1,t} \\ \tilde{y}_t \end{bmatrix} = \boldsymbol{E} \begin{bmatrix} \pi_{H,t-1} \\ \xi_{1,t-1} \\ \tilde{y}_{t-1} \end{bmatrix} \quad \text{for } \; t = t_C + 2, t_C + 3, ... \tag{47}$$

for some real matrix $\boldsymbol{E}$.

Let's now focus on the equilibrium allocation for period $t_C + 1$. This is the first period after $t = 0$ that ZLB is conjectured not to bind. Therefore, $\xi_{2,t_C+1} = 0$ and $\xi_{2,t_C} > 0$. Plug this in the FOC and NKPC, we can obtain the equilibrium condition for this period:

$$\pi_{t_C+1} + \xi_{1,t_C+1} - \xi_{1,t_C} - \frac{1}{\beta\sigma}\xi_{2,t_C} = 0$$

$$\lambda_1 \tilde{y}_{t_C+1} - \lambda(1+\varphi)\xi_{1,t_C+1} - \frac{1}{\beta}\xi_{2,t_C} = 0$$

$$\pi_{t_C+1} = \beta\pi_{t_C+2} + \lambda(1+\varphi)\tilde{y}_{t_C+1}$$

Note that from the equilibrium condition (47), we can express π_{t_C+2} as a linear combination of π_{t_C+1}, ξ_{1,t_C+1}, and $\tilde{y}_{t_C+1}$. As a result, the NKPC gives us an expression of ξ_{1,t_C+1} in terms of π_{t_C+1} and $\tilde{y}_{t_C+1}$. That is

$$\begin{aligned} \xi_{1,t_C+1} &= \mathbf{a}\pi_{t_C+1} + \mathbf{b}\tilde{y}_{t_C+1} \\ \xi_{2,t_C+1} &= 0 \end{aligned} \tag{48}$$

for some real number $\mathbf{a}$ and $\mathbf{b}$.

I then substitute ξ_{1,t_C+1} out and the two FOCs are (linear) functions that can be organized as follow:

$$\begin{bmatrix} \pi_{t_C+1} \\ \tilde{y}_{t_C+1} \end{bmatrix} = \boldsymbol{F} \begin{bmatrix} \xi_{1,t_C} \\ \xi_{2,t_C} \end{bmatrix}, \tag{49}$$

for some real matrix $\boldsymbol{F}$.

We now turn our focus on the equilibrium allocation for period $0, ..., t_C$. During $t =$

$0, ..., t_C$, the conjectured equilibrium path for interest rate is $i_t = 0$. Plug this in the dynamic IS curve. The NKPC and the new DIS now form a system of two differential equations with two unknowns:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{H,t+1} \end{bmatrix} - \boldsymbol{B}\epsilon_r, \quad \text{for } t = 0, 1, ..., t_Z \tag{50}$$

and

$$\begin{bmatrix} \tilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{H,t+1} \end{bmatrix} + \boldsymbol{B}\rho, \quad \text{for } t = t_Z + 1, ..., t_C \tag{51}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined before. Moreover, the FOC condition (43) and (44) can be organize as follow:

$$\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} - \boldsymbol{J} \begin{bmatrix} \tilde{y}_t \\ \pi_{H,t} \end{bmatrix}, \quad \text{for } t = 0, ..., t_C, \tag{52}$$

with initial condition $\xi_{1,-1} = \xi_{2,-1} = 0$.

Now we have the equilibrium allocation for all the periods and can solve the system. The equilibrium path can be determined as follows:

1. Take an initial guess of t_C.

2. Given initial condition $\xi_{1,-1} = \xi_{2,-1} = 0$ and initial guess t_C. Equation (48), (49), (50), (51), (52) make up a system of $4(t_C + 2)$ equations. Note that these system of equations have $4(t_C + 2)$ unknowns, namely $\{\tilde{y}_t, \pi_{H,t}, \xi_{1,t}, \xi_{2,t}\}_{t=0}^{t_C+1}$. Therefore we can obtain the solution for $\{\tilde{y}_t, \pi_{H,t}, \xi_{1,t}\}_{t=0}^{t_C+1}$.

3. In previous steps, we obtain $(\tilde{y}_{t_C+1}, \pi_{t_C+1}, \xi_{1,t_C+1})$, plug this in (47), we can therefore obtain the solution for $\{\tilde{y}_t, \pi_{H,t}, \xi_{1,t}\}_{t=t_C+2}^{\infty}$.

4. given the solution for $\{\tilde{y}_t, \pi_{H,t}\}_{t=0}^{\infty}$ is obtained, plug this in the DIS equation

$$i_t = r_t^n + \pi_{H,t+1} + \tilde{y}_{t+1} - \tilde{y}_t$$

and check if indeed we have

$$\begin{cases} i_t = 0 & \text{for } t = 0, ..., t_C \\ i_t > 0 & \text{for } t = t_C + 1, ... \end{cases}$$

If the above conditions are not verified, the procedure is repeated for a different value for t_C.

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